FORCE – HALF ATWOOD MACHINE EXPERIMENT

Introduction:
Newton’s second law says that the net external force, \( F_{\text{net}} \), on a body is equal to the product of the body’s mass, \( M \), times its acceleration, \( a \):

\[
\overrightarrow{F}_{\text{net}} = m \cdot \overrightarrow{a}
\]  

(1)

In this activity you will verify Newton’s 2\(^{nd}\) law by studying the motion of a "half-Atwood’s machine (if both masses were hanging down from the pulley, this would be an Atwood’s machine):"

First we need to do some algebra to see how to relate an observation to the law we wish to study. Drawing free body diagrams for each mass and using Newton’s second law, we can find the acceleration of this system. In the first step we need to determine the net force. To find the net force we add ALL acting forces to the objects 1 and 2. We note that the weight of object 1 is irrelevant as it does not act along the direction of motion. In absence of friction \( F_{W1} \) does not affect the motion. Object 1 and 2 are connected by a string. They thus have to move in unison. The accelerations of object 1 to the right and of object 2 down must have the same magnitude:

\[
|\overrightarrow{a}_1| = |\overrightarrow{a}_2| = a
\]  

(2)

The combined mass \( M = M_1 + M_2 \) is moving at the acceleration \( a \). The weight of object 2, \( F_{W2} \), is responsible for the acceleration of the combined system of objects. \( F_{W2} \) is thus the net force acting on the combined system. We use Newton’s 2\(^{nd}\) law:

\[
F_{\text{net}} = F_{W2} = (M_1 + M_2) \cdot a
\]

(3)
to determine the acceleration. The acceleration is the quantity we will be measuring. We solve equation (3) for the acceleration and substitute what we know about \( F_{W2} = -M_2g \):

\[
a = \frac{F_{W2}}{M_1 + M_2} = -\frac{M_2}{M_1 + M_2} \cdot g
\]

How do we assign signs to the different forces? Both Atwood and half Atwood machines perform in fact one dimensional motion. The pulley just folds the direction. In what follows we reverse the direction of the vertical axis such that down is taken as positive. The action of object 2 is down and so it’s positive (all vectors pointing down are now positive). The acceleration is determined by Newton’s 2\textsuperscript{nd} law.

The measured data can be compared to this prediction and serve to confirm or refute its validity:

\[
a = \frac{M_2}{M_1 + M_2} \cdot g
\]

(4)

In this experiment, you will use a "smart pulley" to measure the speed of the system after the hanging mass is released. You will then find the acceleration from the slope of the velocity-time curve.

**Preliminary questions**

1. In this experiment, we will measure the speed \((v)\) of the cart as a function of the time. What should a graph of \(v\) vs. \(t\) look like?

![Graph](image)

2. When I do this experiment with \(M_2 = 50\text{g}\), I find an acceleration of 0.5 m/s\(^2\). If I double \(M_2\) but do not change \(M_1\), what will the acceleration be?

a. 1.0 m/s\(^2\)
b. greater than 1.0 m/s\(^2\)
c. between 0.5 and 1.0 m/s\(^2\)
d. Impossible to determine which of the above is correct without knowing \(M_1\).
**Procedure**

Open Data Studio, select Create Experiment, select the smart pulley and connect it to the interface. Make sure the pulley read the velocity: you can spin the pulley with your hand to show how the speed is registered on a velocity versus time graph.

If you don’t see the velocity on the graph, align the position of the pulley with the photogate (there is a laser beam going across the photogate. The laser beam must pass through the pulley in order for the velocity to be acquired).

Set the track to be horizontal with the use of the bubble level and some sheets of paper.

Start with no masses on the hanger, so the hanging mass is just the hanger itself, 50 (or 100) g. Pull the cart back until the hanging mass is just below the pulley. Click the "start" button, and release the cart. **Be sure to stop the cart before it hits the pulley!**

Now click the stop button.

You should have a plot of $v(t)$ vs. $t$, and it should show that the cart had a constant acceleration after you released it. *How can you tell?*

Measure the acceleration by taking the slope, and record it in the table below.

Now add mass...say, 50 grams... to the hanger and measure the acceleration. Repeat this until you have at least 6 sets of data and make sure to have the two values $M_2 = 100$g and $M_2 = 200$ included (you will need to look at these two values to be able to answer the last question). It might be convenient to record your data in an Excel file, to facilitate the plot

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<th>$M_2$</th>
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**Analysis**

According to equation (4), the acceleration of this system should be proportional to the quantity

$$r = \frac{M_2}{M_1 + M_2}$$

Find the mass of the cart and write down the values of $r$ for each runs in the third column. Record the mass of the cart here:
$M_{\text{cart}} = \underline{\hspace{2cm}}$

Use Excel (or pencil and paper) to plot the acceleration versus $r$. Fit the data with a straight line and find its slope. According to equation (2), this slope should equal $g$. Compare your experimental result to the accepted value. Turn in a printed copy the Excel (or paper) plot.

**Question**

Take a look at your prediction, preliminary question 2. Then look at your data to see if your prediction was right. If not, give the correct answer here and explain why it is the correct answer.