**ROTATIONAL DYNAMICS EXPERIMENT**

**Introduction:**

In this experiment, you will measure the moment of inertia for two different systems: A - disk mounted horizontally, as shown below and B - the same disk with a hoop placed on top of it. The measurements are taken with two different methods called respectively *static* and *dynamic*. Then, you will analyze and compare the results.

**Equipment:** Rotational dynamics apparatus, mass, mass hanger, caliper, pulley, photogate

![Diagram of disk and pulley system]

**Preliminary Questions:**

1. In the figure above, a disk is shown on the platform. If the disk is replaced by a hoop (hollow disk) of the same mass $M$ and radius $R$, describe how the acceleration of the mass $m$ would be effected.

2. If instead, a different value of the mass $m$ is considered, how the acceleration of the system would be effected?

**Procedure**

**STATIC**

Find the moment of inertia by direct measurements of the physical quantities: the mass $M_{\text{disk}}$ and radius $R_{\text{disk}}$ of the solid disk, the mass $M_{\text{hoop}}$ and inside and outside radii ($R_1$ and $R_2$) of the hoop.

$M_{\text{disk}} =$ ____ $M_{\text{hoop}} =$ ____

$R_{\text{disk}} =$ ____ $R_1 =$ ____ $R_2 =$ ____
Calculate the momenta of inertia using the following equations

A - Disk

\[ I^{\text{static}}_A = \frac{1}{2} M_{\text{disk}} R_{\text{disk}}^2 = \quad \]  

B - Disk with Hoop

\[ I^{\text{hoop}}_B = \frac{1}{2} M_{\text{hoop}} \left( R_1^2 + R_2^2 \right) = \quad \]

\[ I^{\text{static}}_B = I^{\text{static}}_A + I^{\text{hoop}}_B = \quad \]

**DYNAMIC**

Each object is mounted so that it rotates about a fixed vertical axis. Its rotational acceleration will be caused by the tension in a string wrapped around a spindle and connected to a hanging mass as shown. Using DataStudio you will measure the linear acceleration of the hanging mass. Draw a freebody diagram for the hanging mass and for the rotating mass system. Be precise indicating where the force are applied on the objects.

Newton's 2nd law applied to the falling mass \( m \) yields

\[ mg - T = ma, \quad \text{or} \quad T = m(g - a), \quad (1) \]

where \( T \) is the tension in the string. Similarly, for the rotating system, if we ignore friction in the spindle bearing

\[ Tr = \frac{Ia}{r}, \quad \text{or} \quad T = \frac{Ia}{r^2}, \quad (2) \]

where \( I \) is the moment of inertia of the rotating system, \( r \) is the spindle radius, and \( Tr \) is the torque applied by the string to the spindle. Combining Eq. (1) and (2) and ignoring the moment of inertia of the ‘smart’ pulley yields

\[ m(g - a) = \frac{Ia}{r^2}. \quad (3) \]

You will use the equation above to find \( I \) by making a linear fit using Excel. Connect the Smart Pulley to the Pasco interface box. Open the file: "RotDynamics.ds" contained in the T:\Datastudio folder. This file opens Data Studio to display a plot of the linear velocity of the falling mass \( m \) versus time. This velocity is measured by the photogate mounted on the smart pulley.
Mount the disk horizontally. Start with a total mass $m = 150$ g at the end of the string (the hanger has mass = 50 g). Allow it to fall and cause the disk to rotate, and record the velocity with Datastudio. Obtain the acceleration by a linear fit of the velocity graph. Repeat using as total mass the values $m = 200$ g, 250 g, 350 g. Calculate also the quantity $m(g-a)$.

### A - Disk

<table>
<thead>
<tr>
<th>$m$</th>
<th>$a$</th>
<th>$m(g-a)$</th>
</tr>
</thead>
<tbody>
<tr>
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</table>

Place the hoop on top of the disk and repeat the measurements in A.

### B - Disk with Hoop

<table>
<thead>
<tr>
<th>$m$</th>
<th>$a$</th>
<th>$m(g-a)$</th>
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### Analysis and questions

1. To find the momentum of inertia $I$, first see that the equation

$$m(g - a) = \frac{I}{r^2}a$$

takes the form of a line when it is written as $y = (slope) \cdot x$ in which $y = m(g-a)$; $(slope) = \frac{I}{r^2}$ and $x = a$. To find the numerical value of the slope enter your data into Excel. Plot $m(g-a)$ versus $a$: $m(g-a)$ on the vertical axis, $a$ on the horizontal axis and label these quantities on the graph. Do a linear fit to the data and display the equation on the plot. **Print** the plot. Notice how the slope is **not** $I$.

A - slope =  

B - slope =
2. Take a measurement of the radius $r$ of the spindle with the use of the caliper

$$r = \text{________}$$

3. Using the value of the slopes and $r$ calculate $I_{\text{dynamic}}$ and enter them in the table below. Enter as well the results found with the static method.

<table>
<thead>
<tr>
<th></th>
<th>A - Disk</th>
<th>B - Disk with Hoop</th>
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</thead>
<tbody>
<tr>
<td>$I^A_{\text{static}}$</td>
<td>=</td>
<td>=</td>
</tr>
<tr>
<td>$I^B_{\text{static}}$</td>
<td>=</td>
<td>=</td>
</tr>
<tr>
<td>$I^A_{\text{dynamic}}$</td>
<td>=</td>
<td>=</td>
</tr>
<tr>
<td>$I^B_{\text{dynamic}}$</td>
<td>=</td>
<td>=</td>
</tr>
</tbody>
</table>

4. Compare your static and dynamic values of $I$ by finding their percentage differences.

\[
\begin{align*}
\text{A} - \quad &\frac{|I_{\text{static}} - I_{\text{dynamic}}|}{(I_{\text{static}} + I_{\text{dynamic}})/2} \times 100 = \quad \text{________}
\end{align*}
\]

\[
\begin{align*}
\text{B} - \quad &\frac{|I_{\text{static}} - I_{\text{dynamic}}|}{(I_{\text{static}} + I_{\text{dynamic}})/2} \times 100 = \quad \text{________}
\end{align*}
\]

5. What factors might account for the % differences between these values? List the possible source of errors for each method.

Static:

Dynamic:

6. How the precision of your results can be improved for each one of the two methods?

Static:

Dynamic:

7. Review your answers to the preliminary questions, and explain any changes you wish to make to them after having done this experiment.