**SIMPLE HARMONIC MOTION EXPERIMENT**

**Introduction:**

In this experiment you will measure the spring constant using two different methods and compare your results. Hooke’s law for a spring states that:

\[ F = -kx, \quad (1) \]

where \( x \) is the displacement of the spring from equilibrium, \( F \) is the force exerted by the spring, and \( k \) is the spring constant. The negative sign just means that the restoring force is opposite in direction to the displacement.

If a spring obeys Hooke’s law, then a mass attached to it moves in a simple harmonic motion when displaced from equilibrium and released. That is,

\[ x = A \cos(\omega t + \phi), \quad (2) \]

where \( A \) is the amplitude of oscillation (maximum displacement from equilibrium), \( \omega \) is the angular frequency in rad/s, and \( \phi \) is a phase angle that depends on when timing starts. \( \omega \) is related to the frequency in hertz (f) and the period (T) by \( \omega = 2\pi f = 2\pi T \). By substituting Eq. (2) into Eq. (1) and using Newton’s 2\text{nd} law of motion, it can be shown that

\[ \omega = \sqrt{\frac{k}{m}}, \quad \text{and} \quad T = 2\pi \sqrt{\frac{m}{k}}. \quad (3) \]

Thus, \( k \) can be measured statically using Eq. (1) or dynamically using Eq. (3).

**Equipment:** vertical long rod, clamp, string, masses, mass hanger, meter stick, force sensor

**Preliminary Questions:**

1. Which will have a longer period of oscillation \( T \), a mass of 0.6 kg or a mass of 0.7 kg (same spring)?

2. Sketch a graph of \( x \) vs. \( t \), from Eq. 2, for the two cases \( \phi = 0 \) and \( \phi = \pi/2 \).

3. Sketch graphs of \( v \) vs. \( t \) and \( a \) vs. \( t \) for the two cases drawn in problem 2.
4. According to Eq. 2, \( x \) varies from \((-A)\) to \((+A)\). At which location(s) does the mass have its greatest speed?

(a) \( x = \pm A \)  \hspace{1cm} (b) \( x = 0 \)  \hspace{1cm} (c) \( 0 < x < A \)  \hspace{1cm} (d) same at all points.

5. At which location(s) does the mass have its greatest potential energy?

(a) \( x = \pm A \)  \hspace{1cm} (b) \( x = 0 \)  \hspace{1cm} (c) \( 0 < x < A \)  \hspace{1cm} (d) same at all points.

Procedure

PART 1

Hang a mass holder from the end of the spring and measure the displacement from this position for added masses \( m = 0.1 \text{ kg}, 0.2 \text{ kg}, \ldots, 0.5 \text{ kg} \) (or 5 values of \( m \) appropriate to your spring). Plot \( F (= mg) \) versus \( x \) and determine \( k_1 \) from the slope of the line. Submit your graph with your report.

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<tr>
<th>( m )</th>
<th>displacement</th>
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Value of \( k_1 = \)_______

PART 2

Hang the spring and mass holder from the force sensor. Connect the force sensor to the Pasco interface. Open the file: "SHM.ds" contained in the T:\Datastudio folder. Beginning with \( m = 0.1 \text{ kg} \), displace the mass from equilibrium and measure force versus time. Fit the data to a sine wave and determine the period \( T \) from your fit parameters. Repeat for \( m = 0.2 \text{ kg}, \ldots, 0.5 \text{ kg} \). From Eq. (3), squaring both side we obtains:

\[
T^2 = \frac{4\pi^2 m}{k_2}
\]

Now plot \( T^2 \) (on the y-axis) versus \( m \) (on the x-axis). Be sure to include the mass of the mass holder. Find the slope and determine \( k_2 \) using \( \text{slope} = (4\pi^2/k_2) \).

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<tr>
<th>( m )</th>
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\( \text{Slope:} \)_______
Submit a sample fit to the sine wave, your table of values for $T$ and $m$, and your graph of $T^2$ vs. $m$ with your report.

Value of $k_2 = ______$

Questions

1. Compare the values of $k$ determined in parts 1 and 2. Are they in good agreement? Calculate their percentage difference

$$\frac{|k_1 - k_2|}{\frac{1}{2}(k_1 + k_2)} \times 100\% = \text{_______}$$

2. What is the meaning of the $y$-axis intercept in your straight line fit in part 2? From equation (3), if $m$ is = 0 then $T$ should be zero, but as you see from your fit it’s not, why?

3. If we did this experiment on the moon (where $g = 1.6 \text{ m/s}^2$), what effect, if anything, would this have on the two parts of this experiment?

PART 1
Effects on the measured forces $F$

Effects on the measured displacement $x$

Effect on the value $k_1$

PART 2
Effects on the mass $m$

Effects on the measured period $T$

Effect on the value $k_2$