$\boxed{\textbf{PH354 - HW2}}$ Tensors

SHOW all your works. Put the answers in a BOX NAME:____________________________

1 Two of the following change of coordinates correspond to a Lorentz transformation, identify both:

A- a boost in space
B- a translation in spacetime
C- a rotation in space
D- a rotation in spacetime
E- a translation in space.

2 Given the following components of the four-vector A :

$$A^\mu = (-2, 3, 1, -1)$$

Compute its components A'^μ after the Lorentz boost $v_x = 0.91c$

3 Write how the following tensorial quantities transform after a Lorentz transformation. Use the appropriate Lorentz Λ matrix for each index.

$$A_\mu$$
$$B^{\mu\nu}$$
$$C^\mu{}_\nu$$
$$D^{\mu\nu}{}_\rho$$
$$E^\mu D_\mu$$
$$F^\mu G_\mu H^\rho$$

4 A vector field as components

$$A^i = (-y, 2, 3x,)$$

Find the components in the new coordinates system given by $x' = xz, y' = y^2 - 2x, z' = -x^2 y + z$

5 Given the scalar function

$$\phi(x) = ln(x) + x^2$$

and the change of coordinate $x' = \ln(x^3)$.
5.1 Find the expression of $\phi'(x')$.
5.2 Show that $\phi(x) = \phi'(x')$ for the given point $x = 2$.

6 Perform the explicit matrix multiplications of the component of the Lorentz matrix with its inverse to prove that $\Lambda^{-1}\Lambda = \mathbb{1}_{4\times 4}$

8 Write down explicitly all the terms in the sums of the 2D expression $A^2{}_1 = B^2{}_\alpha C_1{}^\beta D^\alpha{}_\beta$