PH354 - HW3 The metric

SHOW all your works. Put the answers in a BOX NAME:____________________

1 Given the following components of the two Lorentz vectors A and B:

$$A^\mu = (-2, 0, 0, 1) \qquad B^\mu = (5, 0, 3, 4)$$

1.1 Compute $A - 5B$
1.2 Compute AB
1.3 Calculate the norm of A and B and specify if it is timelike, lightlike or spacelike.

2 Given the Euclidean metric δ_{ij} in Cartesian coordinates (x, y), find its expression in the new coordinates $x' = 3x, y' = 2y$.

3 Find the length of the curve

$$x(\lambda) = 2\lambda \quad y(\lambda) = -\lambda^3 \qquad 0 < \lambda < 1/2$$

on a two dimensional space with metric

$$\eta_{ij} = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$$

You can use software to evaluate the integration.

4 A the vector field has components $A^i = (z^2, x, -1)$ and the metric tensor is:

$$g_{ij} = \begin{pmatrix} y & 3x & 0 \\ 3x & z^2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

Find at the point $P = (1, 0, -2)$ the magnitude of A^i.

5 Given the two-dimensional Minkowski metric

$$\eta_{ij} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Find four two-tensors $A^\mu, B^\mu, C^\mu, D^\mu$ such that each one is lightlike, has all components non-zero and points in a unique direction. Draw the tensors on a spacetime diagram.

6 Given the Euclidean metric δ_{ij} in Cartesian coordinates (x, y), find its expression in the polar coordinates (r, θ). Show your step by step calculations: find the Jacobian and use the indices sum.