**PH354 - HW4** Special Relativity 2

SHOW all your works. Put the answers in a BOX NAME:_____

**1** The equations of motion of a point particle is:

$$\begin{aligned} x^0(\tau) &= \alpha(\tau - 1) \\ x^1(\tau) &= \alpha\tau^2 \end{aligned}$$

find the value of $\tau$ which corresponds to $v = c$.
Hint: start by calculating the $d/d\tau$ of the two expressions above.

**2** Show that $\gamma m v^2 + \gamma^{-1} m c^2 = \gamma m c^2$.

**3** A particle moves along the x-axis with 3-velocity (in natural units):

$$\frac{dx}{dt} = \frac{kt}{\sqrt{1 + k^2 t^2}} \qquad k = constant$$

3.1 Calculate the components of the four velocity.
3.2 Give the expression of the proper time elapsed from $t_i = 0$ to $t_f = t$.
(Hint: integrate $d\tau = \gamma^{-1} dt$, you can use software for the integration).
3.3 Does the particle 3-speed ever exceed the speed of light?

**4** An electron is moving with kinetic energy of 1.264 Mev. What is its speed? (unit of $c$)

**5** A particle has relativistic momentum 817 MeV/c and energy of 1125 MeV for observer $O$.
5.1 What is its rest energy for $O$? (MeV)
5.2 Observer $O'$ in a different frame measures the momentum of the same particle to be 953 Mev/c. What is the corresponding energy of the particle for $O'$? (MeV)

**6** Electrons are accelerated to high speed by a two stages machine. The first stage accelerates the electron from rest to 0.99 $c$. The second from 0.99 $c$ to 0.999 $c$.
6.1 How much energy is needed to accelerate the electron in the first stage? (MeV)
6.2 for the second? (MeV)
6.3 for the electron to reach the speed $c$?

**7** A particle of mass $M$ decays at rest into two particles of equal mass $m$. Find the expression for the speed of each particle as a function of $c, M, m$.

**8** (extra credit). Observer $O$ measures a particle of mass $m$ moving in the $x$ direction to have speed $v$, energy $E$ and momentum $p$. An observer $O'$, moving at speed $v_T$ in the $x$ direction, measure $v', E'$, and $p'$.
8.1 Find $E', p'$ expressed in terms of $m, v_T$, and $v$.
8.2 Prove, using the expressions found in 8.1, that $p'^{\mu} p'_{\mu}$ is an invariant quantity.