

SHOW all your works. Put the answers in a BOX

NAME: _____

1 Prove that the partial derivative of a tensor $\partial_\mu A^\nu$ is not a tensor under a general change of coordinates.

2 The tangent vector A on S^2 has components $A^\theta = \sin(\theta)$, $A^\varphi = \sin^2(\theta)$. Calculate all four components of its covariant derivative.

3 Show that the equator ($\theta = \frac{\pi}{2}$) and that any meridian ($\varphi = \text{constant}$) are the geodesics of S^2 .

4 The invariant line element for the two-sphere S^2 of radius R in spherical coordinate is given by:

$$ds^2 = R^2 d\theta^2 + R^2 \sin^2 \theta d\phi^2$$

7.1 Find the components of $g^{\mu\nu}$.

7.2 Use the metric above to prove that the area of half surface of the sphere is $2\pi R^2$.

5 Show the calculation to obtain all components of the connection Γ^i_{jk} for the metric $ds^2 = d\theta^2 + \sin^2(\theta)d\phi^2$ on the unit sphere S^2 (radius $R = 1$).

6 Which of the following 2D manifolds has non-zero intrinsic curvature?

Torus, Sphere, Cylinder, Mobius strip, Klein bottle, Projective plane, Two-holed torus, Hyperbolic plane.