PH354 - HW6 General Relativity

1 The equivalence principle implies two of the following:

A- equivalence between a free falling frame and inertial frame of reference.
B- equivalence between mass and energy.
C- equivalence between staying on a surface's planet and accelerating frame in empty space.
D- equivalence between a free falling frame and an accelerating frame in empty space.
E- equivalence between staying on a surface's planet and an inertial frame of reference.

2 The following tensorial equations holds in special relativity, write the corresponding equations in the presence of gravity. α, β are constants.

$\alpha \partial_\mu B^{\mu\nu} = \beta \eta_{\rho\sigma} C^{\rho\sigma\nu}$
$C_{\mu\nu} B^\nu = \alpha \partial_\mu f(x)$
$\partial^\mu \partial_\mu A^\nu = \eta^{\nu\rho} K_\rho$

3 Do a dimensional analysis of both sides of the Einstein equation: what are the units of it? (do not assume natural unit).

4 The only independent non-zero component of the Riemann tensor for S^2 is $R_{\theta\phi\theta\phi} = r^2 \sin^2\theta$ where r is the radius of S^2. Starting from the Riemann tensor derive $R = 2/r^2$.

5 Prove that the Einstein equation in vacuum ($T_{\mu\nu} = 0$) can be written as $R_{\mu\nu} = 0$.

6 Give an example of each of the following cases:
6.1 Metric $= \eta_{\mu\nu}$, connection coefficients $= 0$, Riemann tensor $\neq 0$.
6.2 Ricci tensor and Ricci scalar both $= 0$, metric $\neq \eta_{\mu\nu}$.
6.3 Connection coefficients $\neq 0$, Riemann tensor $= 0$.