

SHOW all your works. Put the answers in a BOX

NAME: _____

1 An observer at infinity sees a pulse of light moving with speed $0.8 c$ radially toward a black hole of mass $M = M_{SUN}$

1.1 What is the value of the radial coordinate of pulse?

1.2 How far is the pulse from the event horizon? (calculate the proper distance the pulse of light must travel to reach r_S).

2 Alice is at rest at $r_A = 2 r_{SW}$ from a black hole and sends a light signal to Bob every 9 seconds. Bob is also at rest at r_B , and receives that signals every 11 seconds. What is r_B ? (unit of r_{SW})

3 The mass of the Earth is $M_E = 5.97 \times 10^{24}$ kg.

3.1 Calculate the Schwarzschild radius for the Earth.

3.2 Calculate the corresponding mass density ρ_E of the "Earth black hole".

4 Alice is in free fall toward a black hole. What gravitational effects does she observe as she crosses the event horizon of the black hole? Explain your answer in a few words.

A - very high speed.

B - dilation of the time intervals measured by her clock.

C - contraction of the time intervals measured by her clock.

D - very strong gravitational field.

E - none.

5 The energy momentum tensor for a perfect fluid is $T_{\mu\nu} = (\rho_0 + P/c^2)u_\mu u_\nu - P g_{\mu\nu}$ where ρ_0 is the proper density and P the pressure. Find the components of $T^{\mu\nu}$ for an observer at rest with the fluid. Use spherical coordinates for the Robertson-Walker metric

6 Find the value of ω which gives an Universe that is expanding at constant rate.

7 In a toy model of a Universe $a(t) = \alpha t^2$ where α is a constant. A galaxy is measured to have recession velocity = $0.3 c$ at $t = 5$ years. What is the recession velocity of that galaxy at $t = 20$ years? (answer in unit of c).