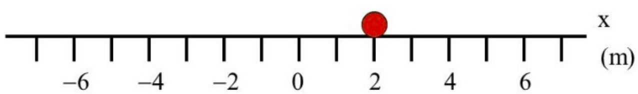
One Dimensional Motion - PH101-

We start physics with *Kinematic:* the study of motion and the relations between the physical quantities necessary to describe it. The quantities are *position*, *velocity* and *acceleration.*

Position

In one dimension the position of an object is given by the value of its coordinate x on the x-axis.



The red sphere is located at the potion x = 2 m

Displacement and Distance

The *displacement* Δx is the change in position.

$$\Delta x = x_f - x_i$$

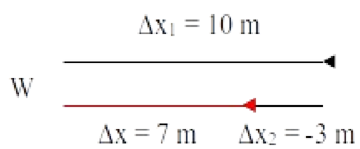
where x_f and x_i are the final and initial positions. In 1-D the displacement can be positive or negative. If the object starts to move at x_i = 5 m and stops and x_f = 3 m then Δx = -2 m is negative, corresponding an object moving to the left on the x-axis.

The *distance D* is a measure of "how much ground an object has covered", is the length of the path. The distance is always non-negative.

Example

An object travels east for 10 m and then west for 3 m.

Total displacement is $\Delta x = \Delta x_1 + \Delta x_2$ = 10 m – 3 m = 7 m. . The distance is D = 10 m + 3 m = 13 m.



Velocity

The velocity indicates how fast a body is moving *and* in which direction is going. The speed represents how fast something is going without notion of direction.

The *average velocity* is the displacement divided by the time interval.

$\bar{v} = \frac{\Delta x}{\Delta t}$ (the bar over v means 'average').

The *average speed* is the distance D divided by the time interval

$$\bar{s} = \frac{D}{\Delta t}$$

Example

In the above example, assume that it takes 3 s to go 10 m east and 2 s to come back 3 m west. Calculate the average velocity and the average speed.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{7m}{5s} = 1.4\ m/s \quad \text{and} \quad \bar{s} = \frac{D}{\Delta t} = \frac{13m}{5s} = 2.6 m/s$$

Note: the average velocity is not equal to the average speed.

The equation of motion

The motion of an object in one dimension is described by the change of its position in time t along the the x-axis. The function *x(t)* is referred as *the equation of motion*. The equation of motion *x(t)* contains all the information about an object's motion. To describe the motion, others kinematic quantities are obtained from *x(t).*

Example

Describe the motion given by the following equations of motion:

x(t) = 3m	the object is not moving since its position does not change.
x(t) = 2(m/s) t	at t = 0 the object is at the origin *O*, as *t* increases it moves away from *O*

The *instantaneous velocity* v (or simply the *velocity*) is the average velocity determined for an infinitesimally short time interval dt. The magnitude of the instantaneous velocity is precisely what you see on yous car speedometer. To determine the instantaneous velocity the let's use an example

The position of a runner is given as $x(t) = ct^2$, where $c = 1$ m/s^2. What is the instantaneous velocity at $t = 1$ s?

You can estimate v by calculating the average velocity during a time interval near $t = 1$ s using smaller and smaller time intervals. From $x(t) = ct^2$, the following values of position versus time can be calculated.

t (s)	x (m)
1.00	1.00
1.01	1.02
1.10	1.21
1.20	1.44
1.50	2.25
2.00	4.00
3.00	9.00

Time interval (s)	Δt (s)	Δx (m)	Δv (m/s)
1.00 to 3.00	2.00	8.00	4.00
1.00 to 2.00	1.00	3.00	3.00
1.00 to 1.50	0.50	1.25	2.50
1.00 to 1.20	0.20	0.44	2.20
1.00 to 1.10	0.10	0.21	2.10
1.00 to 1.01	**0.01**	**0.02**	**2.00**

The table on the left shows the data of the position of the runner at different locations. The table on the right shows the calculate the average velocity over smaller and smaller time intervals.

The calculation in the last row uses the shortest time interval and $v = 2$ m/s is the best estimate of instantaneous velocity at $t = 1.00$ s. Even better estimates can be made by using smaller and smaller time intervals such that the precise definition is obtained by *taking the limit* of Δt to zero. This requires the mathematical notions calculus which are beyond the scope of the course.

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Still, for completeness (this is 105 calculus based stuff you can skip) we have the these definitions and derivations. The *instantaneous velocity* $v$ (or simply *the velocity*) is

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{d}{dt} x(t)$$

the velocity is the derivative of the equation of motion.

For the case $x(t) = ct^2$

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx(t)}{dt} = \frac{d}{dt} ct^2 = 2ct \;\Rightarrow\; v(t=1) = 2\, m/s$$
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and the *instantaneous speed s* (or simply the *speed*) is

$$s = \lim_{\Delta t \to 0} \frac{D}{\Delta t} = \frac{dD}{dt}$$

The magnitude of the infinitesimal displacement |*dx*| equals the infinitesimal distance *dD.*

$$|v| = \lim_{\Delta t \to 0} \frac{|\Delta x|}{\Delta t} = \frac{|dx|}{dt} = \frac{dD}{dt} = s$$

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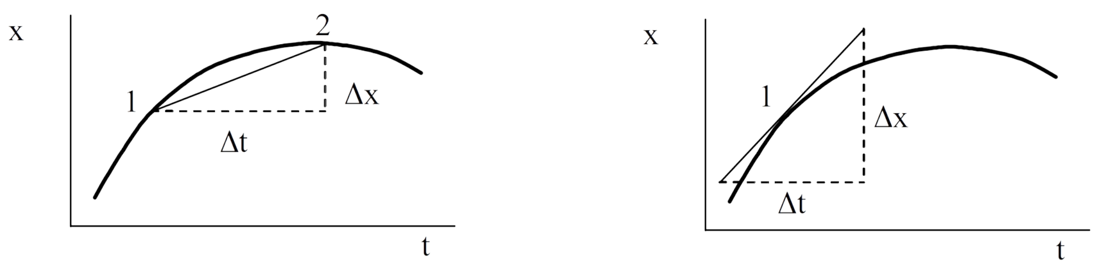
From the 105 considerations above is follows that <u>the magnitude of the velocity equals the speed</u> and it is often refereed as such.

Under many circumstances it is possible to find the values of the instantaneous velocity without the use of calculus. Those are the cases we will considered in this course.

Graphically the instantaneous velocity is the slope of the line tangent of *x(t)* at *t*



Δx/Δt = average velocity from 1 to 2 slope = 2 m/s at *t =1*

Acceleration

Acceleration is the rate at which the velocity changes. In 1-D, it can be positive or negative.

The *average acceleration* is

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

Similar to the case of instantaneous velocity the *instantaneous acceleration* is obtained by taking smaller and smaller time intervals of the average velocity

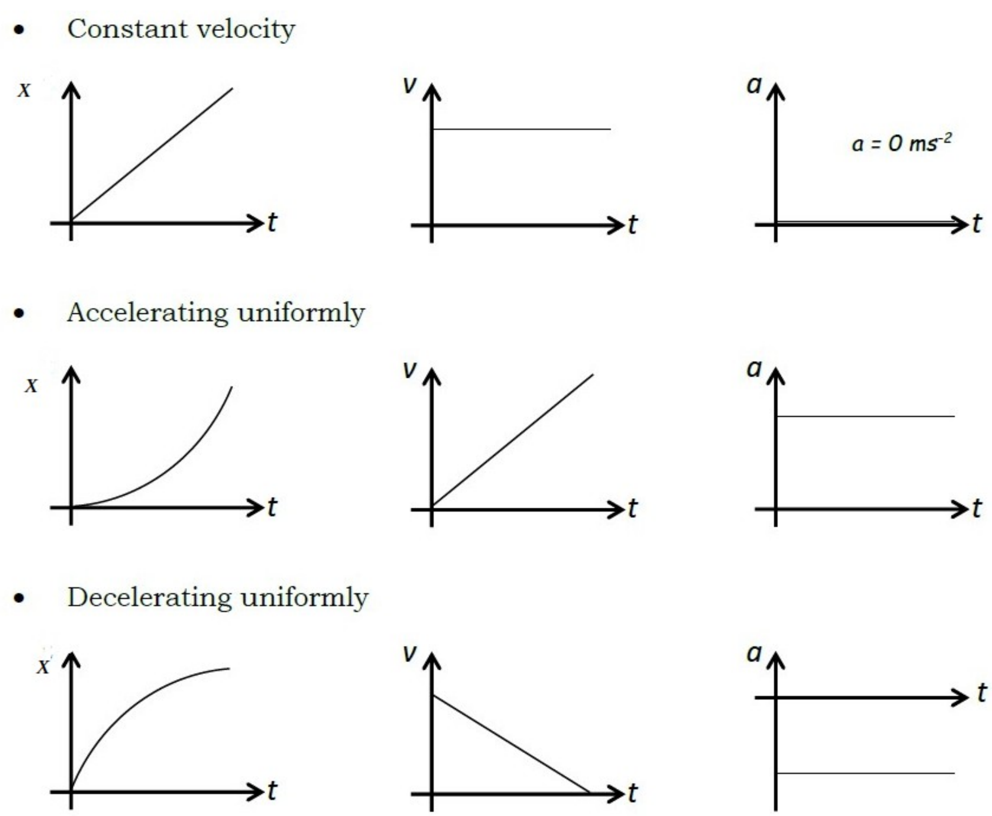
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Graphically the instantaneous acceleration is the slope of the line tangent of *v(t)* at *t*

Motion Graphs

A motion graph is a plot of *x(t)* or *v(t)* or *a(t).* Motion graphs help to visualize the relations between the kinematic quantities *x(t), v(t)* and *a(t).*

Example



Notice how the velocity graph represents the *slope* of the position graph and the acceleration represents the *slope* of the velocity graph. For example if the acceleration is uniform (second case) we have

$$x(t)=c_0 t^2 \Rightarrow v(t)=2c_0 t \Rightarrow a(t)=2c_0 \qquad \text{where } c_0 \text{ is a constant.}$$

How we obtained the two expressions of *v* and *a* from *x(t)* is not important (again calculus), the important point is to being able to related the plots of *x,v* and *a.*

1-D motion at Constant Acceleration

Notation: $x_f = x(t)$, $x_i = x_0$, $\Delta x = x(t) - x_0$, $v_f = v(t)$ $v_i = v_0$
If the acceleration is constant it can be prove that

$$v(t) = at + v_0$$

$$x(t) = \tfrac{1}{2}at^2 + v_0 t + x_0$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v(t)^2 = v_0^2 + 2a\Delta x$$

Example

How long does it take for a car to cross an intersection 30 m wide with a = *2* m/s^2 ?

Given: $x_0 = 0\,m,\ x_f = 30\,m,\ v_0 = 0\,m/s$ and t = ?

$$x(t) = \tfrac{1}{2}at^2 + v_0 t + x_0 \ \Rightarrow\ 30 = \tfrac{1}{2}(2)t^2 + 0 + 0 \ \Rightarrow\ t = \pm\sqrt{2\frac{(30)}{2}} = 5.48\,s$$

Example

A baseball pitch trows a ball with velocity of 44 m/s over a distance of 3.4 m. What is the acceleration of the ball?

Given: $v_0 = 0\,m/s,\ v_f = 44\,m/s,\ \Delta x = 3.5\,m$

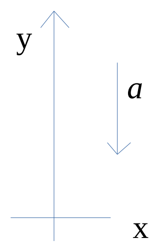
$$v_f^2 = v_0^2 + 2a\Delta x \ \Rightarrow\ a = \frac{v_f^2 - v_i^2}{2a\Delta x} = 280\,m/s^2$$

Free Fall

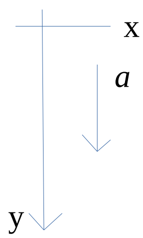
Because of the gravitation attraction of the Earth (we will study more in details gravity in a later chapter) object falls toward the Earth. If the air resistance is neglected (depending on the speed and shape of the object) then the object is said to be in *free fall*.
All objects in free fall, regardless of their masses m, accelerates with equal acceleration and, as long as the free fall takes place nearby the Earth' surface, the acceleration is constant and approximately 9.81 m/s^2 in the downward direction.

We can now use the four equations relative to motion at constant acceleration. By changing the notation $x = y$ and depending on the direction chosen for the vertical axis: $a = -g$ or $a = +g$ where $g = 9.81$ m/s^2,

$$\Delta y = v_0 t - \tfrac{1}{2} g t^2$$
$$v(t) = v_o - g t$$
$$\bar{v} = \frac{v_0 + v}{2}$$
$$v(t)^2 = v_0^2 - 2 g \Delta y$$

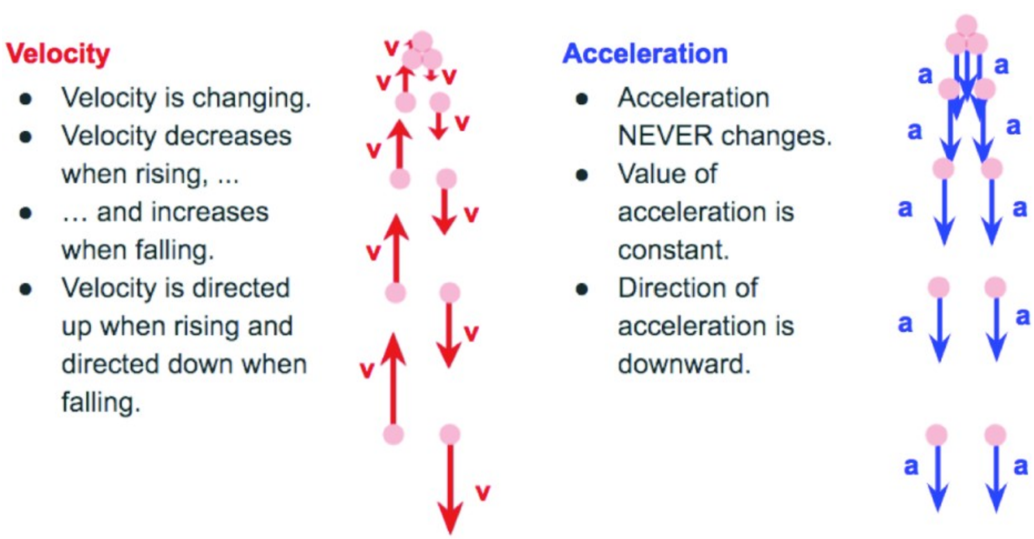


a = -g



a = g

The figure below shows an object that has been thrown up vertically with an initial velocity represented by the red arrow (vector). As it flights up and then back down its velocity changes. Its acceleration instead stays constant throughout the entire motion.



Example

A ball is dropped from rest. How far does it fall in 3 seconds?

$$\Delta y = y + v_0 t - \tfrac{1}{2} g t^2 = -\tfrac{1}{2}(9.8\ \text{m/s}^2)(3\ \text{s})^2 = -44.1\ \text{m}$$

Example

A ball is thrown up with a speed of 20 m/s. How long does it take to reach its peak height?

$$v = v_0 - gt = 0,\quad t = v_0/g = 20/9.8 = 2.04\ \text{s}$$

Example

A ball is thrown up from the edge of a 50 m tall building with a speed of 20 m/s and falls to the street below.

What is the speed of the ball just before it hits the street?

$$v^2 = v_0^2 - 2g\Delta y = (20)^2 - 2(9.8)(-50) = 400 + 980 = 1380$$
$$v = (1380)^{1/2} = 37.1\ \text{m/s}\quad \text{(including direction, } \textit{velocity} = -37.1\ \text{m/s)}$$

How long was the ball in the air?

$$v_{avg} = \tfrac{1}{2}(v_0 + v) = \tfrac{1}{2}(20 - 37.1) = -8.57\ \text{m/s}$$
$$t = \Delta y / v_{avg} = -50/(-8.57) = 5.8\ \text{s}$$