Electromagnetic Waves

Faraday's aw shows that a changing magnetic field produces an electric field.

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}(t)\cdot d\vec{A}$$

In this form, we see that a changing magnetic **B**(t) field produces an electric field. Maxwell's intuition is based on the possibility that a changing electric field could be a source of a magnetic field. In order to achieve this, Maxwell modified the Ampere's law

$$\oint B\cdot ds = \mu_0 I$$

by adding an additional turn to the right-hand side of Ampere's law, so that the complete equation is

$$\oint B\cdot ds = \mu_0 (I + I_{dis}) \qquad \text{(Ampere-Maxwell equation)}$$

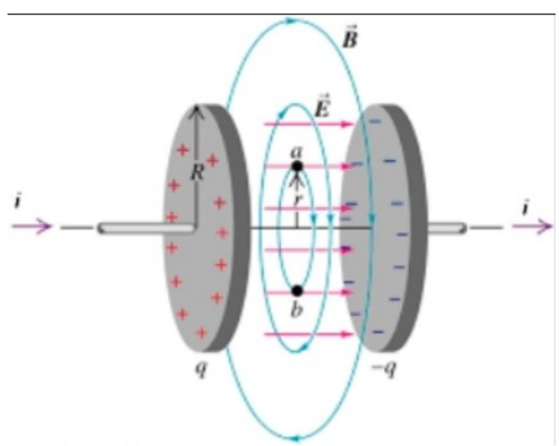
This additional term called the *displacement current* I_{dis} (not to be confused with I_d the induced current) and is given by

$$I_{dis} = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A}$$

The displacement current is an 'effective' current since it not given by a motion of charges. The new term, being proportional to $(\varepsilon_0\ \mu_0) = 1.11 \times 10^{(-17)}$, is extremely small and this explains why no one at the time (or before) Maxwell had ever observed any phenomena related to I_{dis}.
Maxwell's intuition was motivated simply by conceptual considerations.

Experimentally the existence a **B** field generated by I_{dis} can be proved by placing a compass between the plates of capacitor

In the process of charging (or discharging) **E** is generated between the plates. This **E** varies in time and as the variation takes place a compass needle moves indicating a **B** field is present as well.



So, a changing magnetic field produces and electric field and, conversely, a changing electric field produces a magnetic field. Faraday's law and Ampere's law modified to include the displacement current, along with Gauss' law for electric and for magnetism constitute Maxwell's laws of electromagnetism.

The Maxwell's laws of electromagnetism

$$I) \quad \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$II) \quad \oint \vec{B} \cdot d\vec{A} = 0$$

$$III) \quad \oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A}$$

$$IV) \quad \oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d}{dt}\int \vec{E} \cdot d\vec{A}$$

Note that Maxwell's equations are nearly symmetric in **E** and **B**. Since there are apparently no magnetic monopoles analogous the electric charges, then we have zero on the right-hand side of the second equation and no monopole current term on the right-hand side of the third equation.

James C. Maxwell
(1831–1879)

In term of the sources for **E** and **B** we now have the complete picture:

Source of **E**	Source of **B**
Q: electric charge	*I: electric current*
dΦ(B)/dt	*dΦ(E)/dt*

And, because of Ampere's law, magnets are not considered as fundamental sources of ***B***.

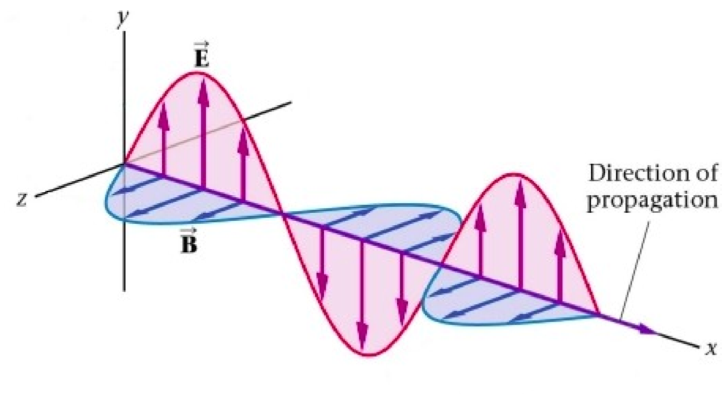
Electromagnetic waves

One of the main result of Maxwell Theory is the prediction of electromagnetic waves. They consist of oscillating electric and magnetic fields that propagate through space at the speed of light. An oscillating **E** produces an oscillation **B**, and vice versa. The existence of electromagnetic waves was proven experimentally Hertz in 1889.

By combing the four equation and setting $Q = I = 0$ (no sources) we obtained two waves equations (one for **E** and one for **B**)

$$\frac{\partial^2 E_y(x,t)}{\partial t^2} - \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 E_y(x,t)}{\partial x^2} = 0$$

$$\frac{\partial^2 B_z(x,t)}{\partial t^2} - \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 B_z(x,t)}{\partial x^2} = 0$$



with solutions

$$E_y(x,t) = E_{max} \sin(kx - \omega t)$$

$$B_z(x,t) = B_{max} \sin(kx - \omega t)$$

where $k = \frac{2\pi}{\lambda}$ is the wave number and $\omega = 2\pi f$ is the angular frequency.

For a fixed frequency, both **E** and **B** vary sinusoidally in time and in space. Both **E** and **B** are perpendicular to the direction of travel of the wave and they are perpendicular to each other.

The two equations above describe an particular solution of a **E** field oscillating in the *y* direction and a **B** field oscillating in the *z* direction. The wave travels in the *x*- direction. From the wave equations above it follows that the *speed* of the electromagnetic waves is

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} .$$

Using the accepted values of $\mu_0 = 4 \times 10^{-7}$ T m/A and $\varepsilon_0 = 8.85 \times 10^{-12}$ $C^2/(N \cdot m^2)$, this equation gives $c = 3 \times 10^8$ m/s, which is the speed of light. This is to be expected since visible light is a part of the electromagnetic spectrum.

It can also be shown that the ratio of the magnitudes of **E** and **B** is fixed as

$$\frac{E}{B} = c$$

Written as $E = cB$ the equation does not imply that $E >> B$ or that the 'amount' of E in the electromagnetic waves is much greater then the amount of B. The field **E** and **B** have different units and so a comparison of their magnitude is not possible. What needs to be compared are their energy densities.

The energy densities u_E and u_B are given by

$$u_E = \frac{1}{2}\epsilon_0 E^2,\ u_B = \frac{1}{2\mu_0}B^2$$

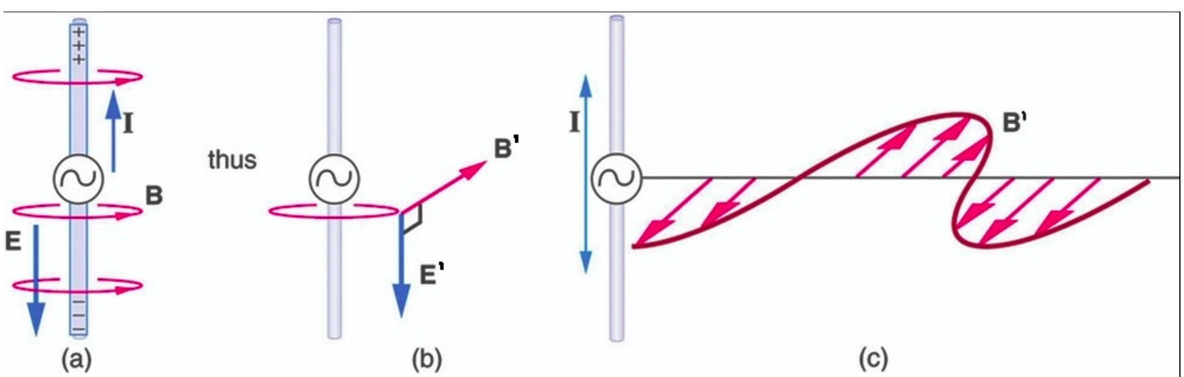
Since $B = E/c$ and $c = 1/\sqrt{\mu_0 \epsilon_0}$ then

$$u_E = \frac{1}{2}\epsilon_0 c^2 B^2 = \frac{1}{2}\epsilon_0\left(\frac{1}{\sqrt{\mu_0 \epsilon_0}}\right)^2 B^2 = \frac{1}{2\mu_0}B^2$$

that is $u_E = u_B$.

Production of electromagnetic waves

Electromagnetic waves can be produced by applying an oscillating potential to an antenna. The antenna could consist of a rod connected to each side of an AC voltage source. The voltage source would generate a sinusoidally varying current and a sinusoidally varying charge distribution in each rod. As a consequence, the rods would generate magnetic and electric fields which would be perpendicular to each other and would radiate from the rods.

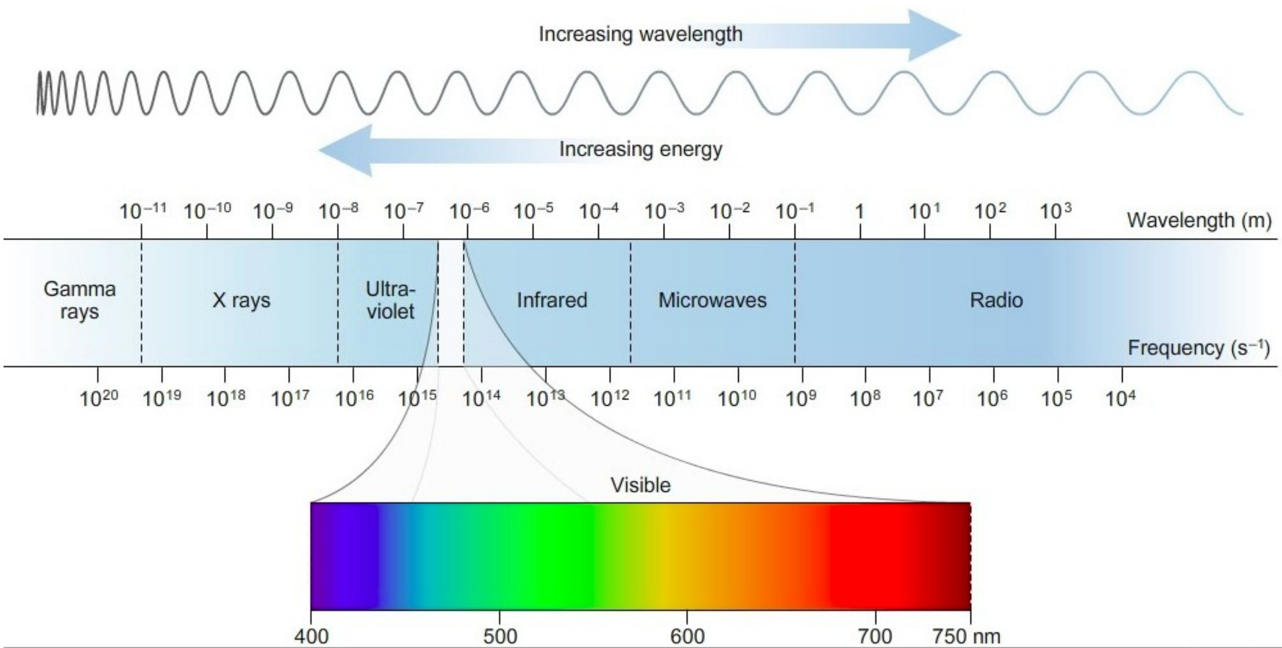


(a) AC current generate both an oscillating **E** and **B** fields.
(b) The oscillating **E** and **B** generates the induced **E'** and **B'** field
(c) The induced fields propagates (only **B'** is shown)

At the most fundamental level, an electromagnetic wave is generated when an electric charge accelerates.

Electromagnetic Spectrum

The speed of an electromagnetic wave is $c = \lambda f$ where λ is the wavelength and f is the frequency. The product λf is constant of nature but electromagnetic waves can have different λ and f. The Electromagnetic Spectrum classifies the various waves



One of main feature of Maxwell theory was to unify in a single framework the nature of different radiations: Gamma ray, X- Ray, Radio Wave, etc, are all made of the same thing: oscillating E and B fields. Their difference is *just* in their wavelengths.

It was very interesting the discovery that visible light is also an electromagnetic wave. Its wavelength range is 400 to 750 nano meters depending on the color. When you are looking at light you are looking at **E** and **B** fields.

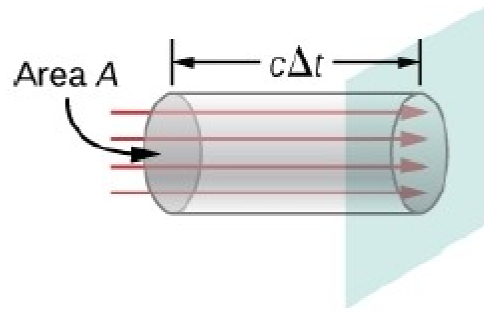
The study of light and its proprieties is called *Optics.* The Maxwell theory of electromagnetism allows for the inclusion of optics into electromagnetism.

Energy carried by electromagnetic waves

The energy density of an electromagnetic wave is the sum of energy densities of the electric and the magnetic field $u = u_E + u_B$. The total energy ΔU contained in a volume V is

$$\Delta U = uV = uA\Delta x$$

Since the wave is moving $\Delta x = c\Delta t$ the flux of energy through a surface A perpendicular to x in a time Δt is



$$\text{flux} = \frac{uAc\Delta t}{A\Delta t} = uc = \epsilon_0 c E^2 = \frac{1}{\mu_0} EB$$

The energy flux has the units of intensity (W/m^2).

In order to describe the energy flux the *Poynting vector* $\vec{S}$ is introduced as

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

The direction of $\vec{S}$ is the direction of motion of the wave. The intensity of the wave is the time-averaged power transmitted per unit area of $\vec{S}$

$$I = \langle S \rangle = \frac{1}{2\mu_0} E_{max} B_{max}$$

The factor of ½ is a result of averaging S time.
Using the relationship between **E** and **B** and the expression for c, the intensity can also be written as

$$I = \frac{E_{max}^2}{2\mu_0 c} = \frac{c}{2\mu_0} B_{max}^2$$

Example

The intensity of sunlight incident upon the Earth is about 1,400 W/m^2. What are the maximum values of the electric and magnetic fields associated with the radiation?

$$I = \frac{E_{\max}^2}{2\mu_0 c}$$

$$E_{\max} = \sqrt{2\mu_0 c I} = \sqrt{2(4\pi x 10^{-7})(3x10^8)(1400)} = 1,027 \ V/m$$

$$B_{\max} = \frac{E_{\max}}{c} = \frac{1027}{3x10^8} = 3.4x10^{-6} T$$

How much solar power is incident upon the Earth?

The effective area of the Earth seen by the Sun is that of a circle with the Earth's radius *R*

$$P = IA = I\pi R^2 = (1400\, W/m^2)\pi(6.38x10^6 m)^2 = 1.79x10^{17} W$$

Point-like sources

When the size of a source of electromagnetic waves is much smaller compared to the distance at which the wave is detected, then the source can be considered a point-like. Examples are the Sun respect to us or a light bulbs as observed from a meter away. We can assume the point source radiates uniformly in all directions. All of the power *P* must pass through a sphere of radius *r* and area $A = 4\pi r^2$. Thus, the intensity at a distance *r* from the source is

$$I = \frac{P}{4\pi r^2}$$

Note how the power is a intrinsic propriety of the source, while the intensity depends on the observer.

Example

What is the power of the Sun, i.e. what is the amount of energy the Sun gives away per second? The power of the Sun is refereed as *brightness* or *luminosity*.

$$Power = I(4\pi r_E^2) = (1400\, W/m^2)4\pi(150\times10^6 km) \approx 3.8\times10^{23} W$$

Where r_E is the average Earth – Sun distance.

Example

A square sheet with side of 10 cm is 5.8 m away from a 120 W light bulb. What is the power absorbed by the sheet?

The intensity of light at the location of the sheet is

$$I = 120\,W/\left(4\,\pi\ x\ (5.8\,m)^2\right) = 0.28\,W/m^2$$

and the power absorbed by the sheet of area *A*= 100 cm^2

$$P = IA = \left(0.28\,W/m^2\right)x\left(10^{-2}m^2\right) = 2.8\,x\,10^{-3}\,W$$

Momentum carried by electromagnetic waves

Electromagnetic waves also carry momentum, even though they don't have mass. If an electromagnetic wave strikes a surface of areas *A* the theory of collisions applies (conservation of total linear momentum) and the momentum of the surface changes.

$$\vec{p}^{\,i}_{TOT} = \vec{p}^{\,f}_{TOT}$$
$$\vec{p}^{\,i}_S + \vec{p}^{\,i}_W = \vec{p}^{\,f}_S + \vec{p}^{\,f}_W$$

where $\vec{p}_S$ and $\vec{p}_W$ indicates the momentum of the surface and the electromagnetic wave. The change of the momentum of the surface $\Delta\,\vec{p}_S = \vec{p}^{\,f}_S - \vec{p}^{\,i}_S$ depends on how much of the incident wave is reflected or absorbed by the surface.

$$\Delta\,\vec{p}_S = \vec{p}^{\,i}_W - \vec{p}^{\,f}_W$$

The energy carries by the incident electromagnetic waves is $\Delta U = IA\Delta t$ and the magnitude of its momentum is $p^i_w = \Delta\,U/c$. If the surface completely absorbs the wave

$$\Delta\,p_S = p^i_W - 0 = \frac{\Delta\,U}{c} \qquad \text{(complete absorption)}$$

If the surface completely reflects the wave then $\vec{p}^{\,f}_W = -\vec{p}^{\,i}_W$ and we have

$$\Delta\,p_S = \left|\vec{p}^{\,i}_W - \left(-\vec{p}^{\,f}_W\right)\right| = 2\frac{\Delta\,U}{c} \qquad \text{(complete reflection)}$$

Radiation force and pressure

When the electromagnetic wave strikes a surface it imparts a force to the surface (since force is rate of change of momentum).
For an object that completely absorbs the radiation the magnitude of the force is

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta U / c}{\Delta t} = \frac{IA}{c} \qquad \text{(complete absorption)}$$

The force is twice as great for a completely reflecting object, so

$$F = \frac{2\,IA}{c} \qquad \text{(complete reflection)}$$

Since the pressure is the force per unit area, there is also a radiation pressure given by

$$Pressure = \frac{I}{c} \qquad \text{(complete absorption)}$$

$$Pressure = \frac{2\,I}{c} \qquad \text{(complete reflection)}$$

Example

If all the Sun's radiation is absorbed by the Earth, then what is the force imparted by this radiation on the Earth?

The power absorbed is the energy absorbed per second. This can be used to find the momentum absorbed per second, which is the force.

$$\Delta p = \frac{\Delta U}{c}$$

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta U / \Delta t}{c} = \frac{P}{c} = \frac{1.79\, x\, 10^{17}\, W}{3\, x\, 10^{8}\, m/s} = 6.0\, x\, 10^{8}\, N$$

Although this is a large force, it has an imperceptible effect due to the large mass of Earth.