Reflection and Refraction of Light

Light is an electromagnetic wave. Visible light is the part of the electromagnetic spectrum with wavelength between about 400 nm (ultraviolet) and 700 nm (red).
In this chapter we will study what happens when a ray of light strikes a surface or travels from one medium to another. We can derived all the results contained in this chapter directly from the Maxwell equations. Instead we will use the `Ray approximation of optics` where we assume that light travels as a straight line (and not as a wave) in a homogeneous medium. This assumption simplifies considerably the theory and works as a valid approximation as long as the wavelengths are small.
In the next chapter we will study instead phenomena which require the wave description of light of Maxwell theory (wave optics).

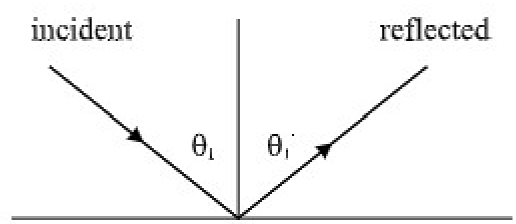
Reflection

When light strikes a surface, part of the light is reflected and - if the surface bounds a transparent medium – part of the light is transmitted. If the surface smooth compared with the wavelength of the light, then a single ray of light incident light will reflect as a single ray. This is referred to as *specular* reflection. If the surface is rough compared with the wavelength of the light, then light will reflect over a range of angles. This is referred to as *diffuse* reflection. In the following we will assume that the surfaces are smooth.

The *law of reflection* states that the angle of reflection is equal to the angle of incidence –

$$\theta_1' = \theta_1$$

The angles are measured from the direction normal (perpendicular) to the surface.

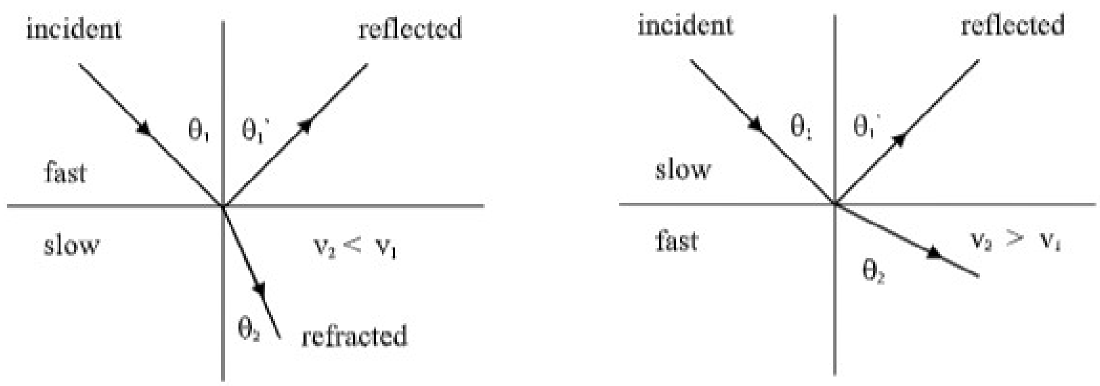


Refraction

Light travels at a speed $c = 3 \times 10^8$ m/s in a vacuum. In a transparent medium, light travels slower than in a vacuum. Refraction refers to the bending of the transmitted light at the interface between two transparent materials. The angle of refraction depends on the angle of incidence and the relative speeds of the light in the two different media and can be found from the relationship

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$$

From this equation we see that the direction in which the light is bent depends on whether it is going from a slower to a faster medium or vice versa.



Index of refraction

The index of refraction of a medium is defined as the ratio of the speed of light in vacuum to the speed of light in the medium.

$$n = \frac{c}{v}$$

By definition, $n = 1$ in vacuum. In all materials $n > 1$.
The frequency of light does not change as it goes from one a medium of one index of refraction to another. This means that the wavelength changes with the speed.

$$f = \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{v_2}{v_1} = \frac{c/n_2}{c/n_1} = \frac{n_1}{n_2}$$

Thus, the wavelength is smaller in a medium than in vacuum.
From the definition of the index of refraction, we can rewrite the relationship between the incident and refracted angles as

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{n_1}{n_2} \quad \text{or}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \text{(Snell's Law)}$$

This equation is known as *Snell's law* of refraction.

Example

The index of refraction of a glass is n = 1.6. A ray of light is incident upon the glass surface at an angle of 30^o. What is the angle of refraction into the glass?

Assume that air surrounds the glass and that the index of refraction of air is approximately n = 1.

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = \left(\frac{1}{1.6}\right)\sin(30^o) = 0.3125$$

$$\theta_2 = 18.2^o$$

If the wavelength of the light in air is 520 nm (green), what is its wavelength in the glass?

$$\frac{\lambda_2}{\lambda_1} = \frac{n_1}{n_2}$$

$$\lambda_2 = \frac{n_1}{n_2}\lambda_1 = \left(\frac{1}{1.6}\right)510\ nm = 319\ nm$$

What is the frequency of the light?

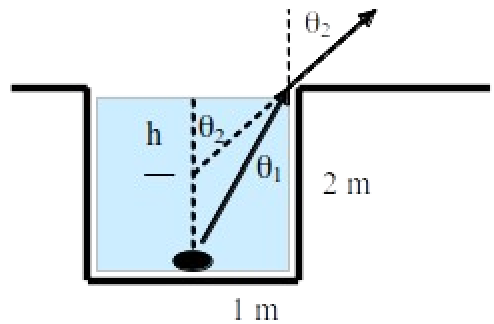
$$f = \frac{v}{\lambda} = \frac{c}{\lambda_1} = \frac{3\,x\,10^8\,m/s}{510\,x\,10^{-9}m} = 5.88\,x\,10^{14}\ Hz$$

Example

A coin is at the bottom of a pool of water 2 m deep and it is 1 m from the side of the pool. If you look into the edge of the pool, how deep does the penny appear to be below the surface? The index of refraction of water is 1.33.

The incident angle is given by

$$\theta_1 = \tan^{-1}\left(\frac{1}{2}\right) = 26.6^o$$

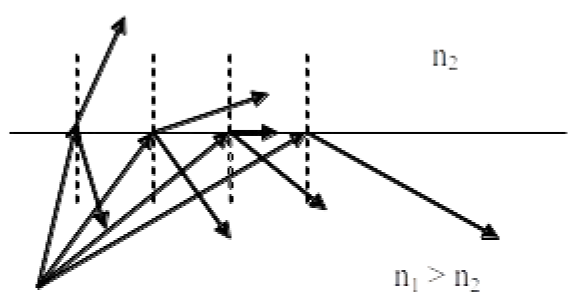


The angle of refraction is

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = \frac{1.33}{1}\sin(26.6^o) = 0.595$$
$$\theta_2 = \sin^{-1}(0.595) = 36.5^o$$
$$\tan\theta_2 = \frac{1}{h}$$
$$h = \frac{1}{\tan(36.5^o)} = 1.35\ m$$

Total internal reflection

If light goes from a slow medium to a fast medium (e.g., from glass into air), then the angle of refraction is greater than the angle of incidence.
This means that the angle of refraction can become 90^o, at which point there is no longer propagation of light into the faster medium. The light will then be totally reflected. The critical angle for total internal reflection is obtained as follows. Assume the light travels from medium 1 to medium 2 and that $n_1 > n_2$. Then the critical angle is $\theta_c = \theta_1$ when $\theta_2 = 90^o$.



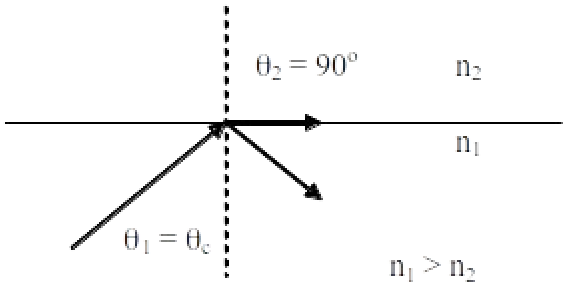
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$n_1 \sin\theta_c = n_2 \sin 90^o = n_2$$

Or,

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \qquad (n_1 > n_2)$$

Example

A man underwater shines a flashlight up towards the surface. At what angle is the light totally reflected back into the water?

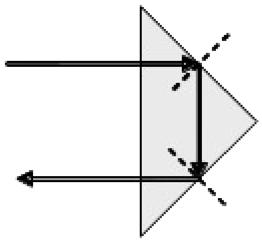


The index of refraction of water is n = 1.33.

$$\theta_c = \sin^{-1}\left(\frac{1}{1.33}\right) = 48.8^o$$

Example

A 45°-45°-90° prism is used to totally reflect light incident upon its large surface. What is the required minimum index of refraction of the prism?

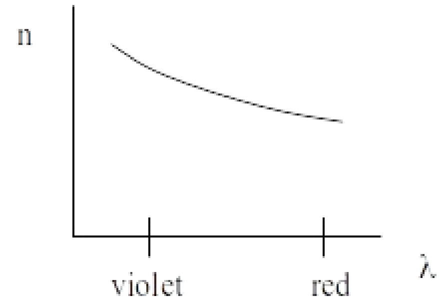


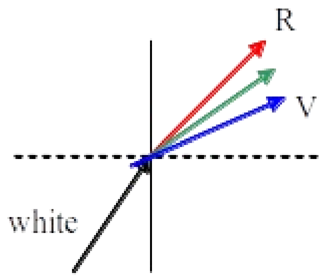
Referring to the figure, the light strikes the back side of the prism at an angle of 45°. Then

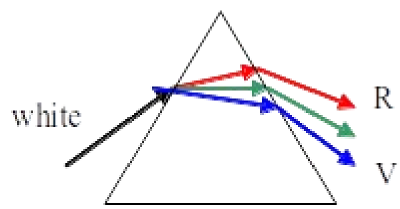
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$n \sin\theta_c = (1)\sin 90^o = 1$$
$$n = \frac{1}{\sin 45^o} = \frac{1}{0.707} = 1.414$$

Dispersion of light

The index of refraction in transparent materials such as glass decreases as the wavelength increase, somewhat as shown in the figure. This means that the short wavelengths bend more at a surface between two transparent materials than the long wavelengths. Thus, white light, which consists of a mix of wavelengths, will be dispersed into its component colors when it bends. Since violet has a shorter wavelength than red, then it will be bent more than the red. If white light is passed through a prism, then it is bent going in and coming out.





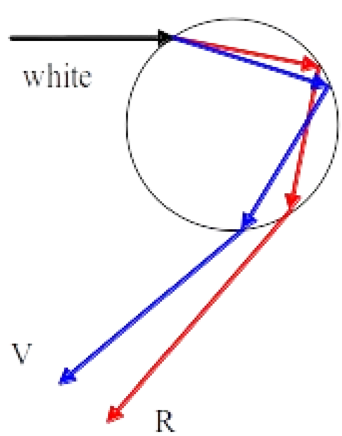


Dispersion of light is also referred as *diffusion*.

Rainbows

Rainbows are caused by the dispersion of white light by water droplets in the sky. Most rainbows are produced as shown in the diagram to the left. The red part of the rainbow comes from the steeper rays and is at the top. In some cases a secondary rainbow can be seen above the primary rainbow with the colors reversed. Note that the primary rainbow if formed by one reflection from the back of the droplets, while the secondary rainbow undergoes two reflections in the droplet.

Primary Rainbow



Secondary Rainbow