Wave Optics

Wave optics accounts for the fact that light is an electromagnetic wave. As such it can undergo interference and diffraction effects, similar to what can be observed for sound waves. The electric and magnetic field vectors associated with light are perpendicular to the direction of propagation. Thus, light is a transverse wave (not like sound in air) and can be polarized.

These effects due to the wave nature of light do not easily occur in Nature since some peculiar conditions need to be satisfied. This requires specific experimental set up and devices which, for example at the time of Newton, did not exist. In other words, Newton did not consider a wave description for light because no observations were known to indicate so.

Interference

Two waves can interfere constructively or destructively, depending on their relative phase. In constructive interference the resultant wave has greater amplitude than either of the individual waves. In destructive interference the resultant wave has a lesser amplitude than either of the individual waves.

The figure on the right shows interference of water waves.

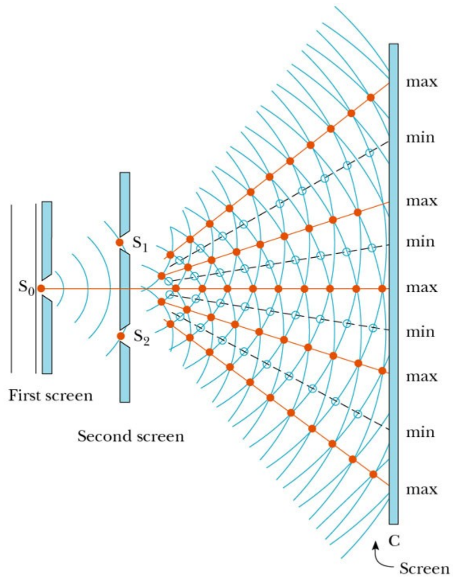
In order to produce and observer the interference of light waves, three conditions must be satisfy:

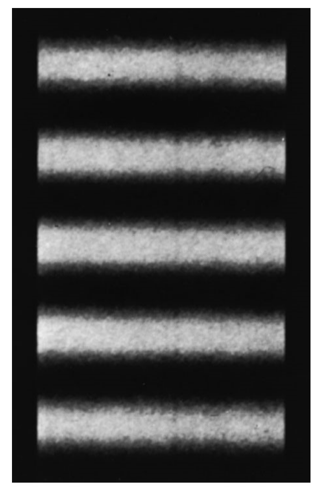
1) the waves must be monochromatic (of the same wavelength).
2) the wavelength must be of about same order of magnitude compare to the *size* of the object light interact with (which depends on the case).
3) the waves must be *coherent.*

Coherent means the waves have a phase difference which stays constant in time. The light from an ordinary light source, even if it has a well-defined wavelength, is incoherent. That is, the phase of the light is only constant for short periods of time.

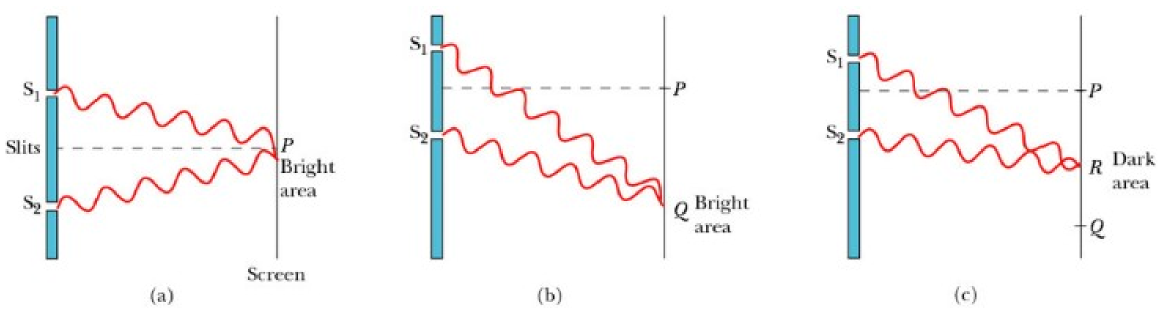
Double Slit

Two coherent light sources can be obtained by shining a single monochromatic light beam onto two closely spaced slits. Thus, if the phase of the light from the source changes, this change is the same at both slits. If the light from these two slits is projected onto a surface, then bright and dark lines can be observed on the surface due to the constructive and destructive interference of the two sources. This type of setup is called Young's double slit experiment (named after Thomas Young, who first demonstrated this type of interference in 1801). The setup is shown below.





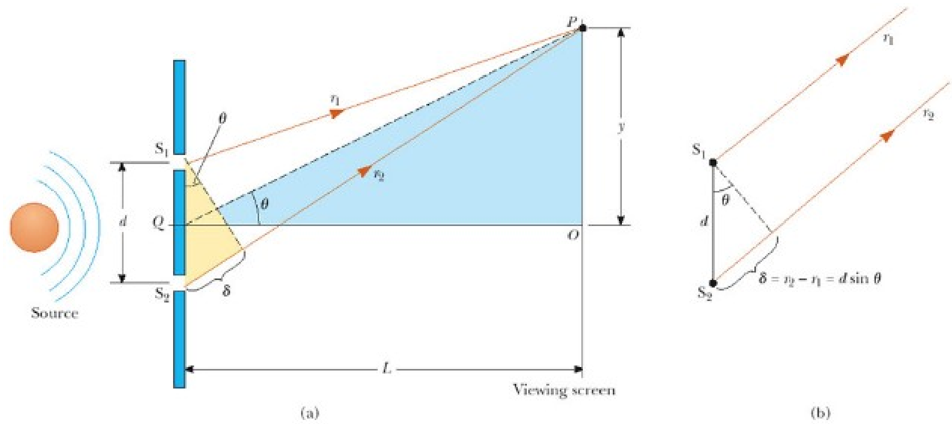
In the center of the screen the light coming from the two slits travels the same distance. They are thus in phase and add constructively, giving a bright line. Above and below the center line the light arriving from the two slits travel different distances and will be either in phase or out of phase depending on the path difference. If the path difference is an integral number of wavelengths, then the light waves are in phase, they add constructively, and a bright line is observed. If the path difference is a half-integral number of wavelengths, they are out of phase, they add destructively, and a dark region is observed. As a consequence, nearly equally spaced fringes are observed on the screen.



The distance L between the slits and the screen is typically large compared with the distance d between the slits, so that the light waves from the two slits to a point on the screen are essentially parallel. In this case, as can be seen from the construction below, the path difference between the two waves is given by

$$\delta = r_2 - r_1 = d \sin\theta \ ,$$

where θ is the angle between the normal to the slit plane and a line from the slits to the point on the screen where interference is observed.



For constructive interference (bright lines), the path difference must be an integral number of wavelengths. That is,

$$m\lambda = d \sin\theta, \ \ m = 0, \pm 1, \pm 2, \pm 3, \ldots \qquad \text{(bright)}$$

For destructive interference (dark lines), the path difference must be a half integral number of wavelengths, or

$$\left(m + \frac{1}{2}\right)\lambda = d \sin\theta, \ \ m = 0, \pm 1, \pm 2, \pm 3, \ldots \qquad \text{(dark)}$$

The above expressions giving the angular positions of the bright and dark fringes can be used to give the vertical positions y of the fringes on the screen. From the previous figure, we see that

$$\tan\theta = \frac{y}{L} \ .$$

If the angles are not very large, which is usually the case, then *tan* $\theta \approx$ *sin* θ. Then for the bright fringes,

$$y_m = \frac{\lambda L}{d} m, \ \ m = 0, \pm 1, \pm 2, \pm 3, \ldots \qquad \text{(bright)}$$

Likewise, for the dark fringes,

$$y_m = \frac{\lambda L}{d}\left(m+\frac{1}{2}\right),\ m = 0, \pm 1, \pm 2, \ldots \qquad \text{(dark)}$$

The separation of the fringes is given by

$$\Delta y \;=\; y_{m+1} - y_m \;=\; \frac{\lambda L}{d}$$

Note from this equation how λ needs to be of the same order of d as requested by the condition 2) where now d represents the *size* of the object (see condition 2 for interference to occur). If $d >> \lambda$ then Δy goes to zero and the interference patter disappears.

Example

A double-slit interference pattern is obtained using a light source with wavelength 650 nm and two slits with center-to-center separation 0.1 mm. The distance from the slits to the screen is 2 m. What is the separation of the bright fringes on the screen?

$$\Delta y = \frac{\lambda L}{d} = \frac{(650\,x\,10^{-9}\,m)(2\,m)}{0.1\,x\,10^{-3}\,m} = 0.013\ m$$

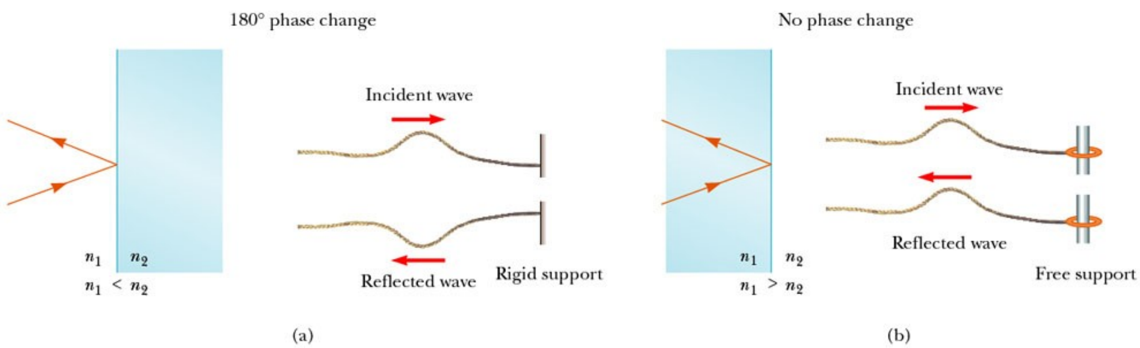
What is the angular position of the 3rd bright fringe from the center fringe?

$$\sin\theta = \frac{m\lambda}{d} = \frac{(3)(650\times 10^{-9}\,m)}{(0.1\times 10^{-3}\,m)} = 0.0195$$
$$\theta = 1.1^{o}$$

Phase Change due to Reflection

Interference effects can be observed because of reflection of a monochromatic light source from the interface between two transparent materials. In order to determine the nature of the interference, we need to know what happens to the phase of the light upon reflection. When light traveling in a medium with index of refraction n_1 is traveling toward a medium with index of refraction n_2, then some of the light is reflected at the interface. If $n_1 < n_2$, then the phase of the reflected light changes by 180°. However, if $n_1 > n_2$, then there is no phase change upon reflection.

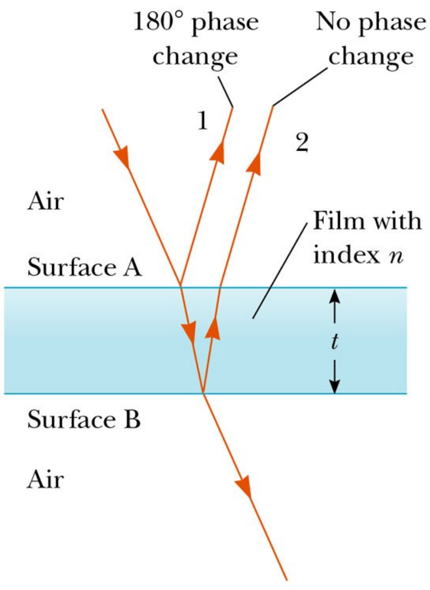
A mechanical analogy for this phase change is a wave traveling on a string. If the string is rigidly attached to a support, then the reflected wave is inverted. That is, its phase changes by 180°. On the other hand, if the end of the string is loosely attached, then the reflected wave is not inverted.



A similar situation occurs if a light rope and a heavy rope are tied together. There will be reflection at the junction of the ropes. If the wave travels from the light rope toward the heavy rope (fast to slow), then the reflected wave will be inverted. If it travels from the heavy rope to the light rope (slow to fast), then the reflected wave will not be inverted.

Thin Film Interference

A light wave incident upon a thin transparent film will be reflected from both the top surface and the bottom surface. These two reflected waves can interfere constructively or destructively, depending on their phase difference. The phase difference will be caused by their path length difference (as in double slit interference) and possible differences in phase change caused by the reflections. If the film has index of refraction $n > 1$ and is in air (above and below the film), then a phase change of 180° will occur during the reflection from the top



surface and no phase change will occur during the reflection from the bottom surface. For normal incidence, the path length difference between the two reflected waves is *2t*, where *t* is the film thickness. A phase change of 180° is equivalent to the phase change that comes from a path length difference of one-half wavelength. Thus, in order for these two reflected waves to be in phase and interfere constructively, we must have

$$2t = \left(m+\tfrac{1}{2}\right)\lambda_n \ ,$$

where $\lambda_n = \lambda / n$. With this latter substitution we can write

$$2nt = \left(m+\tfrac{1}{2}\right)\lambda, \ \ m=0, \pm 1, \pm 2, .. \qquad \text{(constructive reflection)}$$

Destructive reflection will occur if

$$2nt = m\lambda, \ \ m=0, \pm 1, \pm 2, .. \qquad \text{(destructive reflection)}$$

If the materials above or below the film are such that the same phase change occurs upon reflection from both surfaces, then the two above equations for constructive and destructive reflection will be reversed.

From the two equations of constructive and destructive reflection, the condition for interference to occur 2) is satisfied when t is of the some order of magnitude of the wavelength, i.e. t represents the size of the object (D = t).

Example

A soap bubble film has thickness *t* = 100 nm. What wavelength in the visible would be most strongly reflected?

Since the film is water, its index of refraction is 1.33.

$$\lambda = \frac{2nt}{m+\frac{1}{2}}$$

$$\lambda_0 = \frac{2nt}{0+\frac{1}{2}} = 4nt = 4(1.33)(100\,nm) = 532\ nm \ \ (green)$$

For higher values of *m*, the *λ* values are in the ultraviolet part of the spectrum and are not visible.

Example

A glass lens with $n = 1.6$ is coated with a thin film of MgF_2 for which $n = 1.38$. What is the minimum thickness of the film such that light with wavelength 550 nm is most weakly reflected?

A phase change of 180° will occur at both the air-MgF_2 surface and the MgF_2-glass surface since in both cases the light is traveling from a material with lower index of refraction to a material with higher index of refraction. Thus, the path length difference (*2t*) must be a half-integral number of wavelengths to get destructive interference. So,

$$\lambda = \frac{2nt}{0 + \frac{1}{2}} = 4nt,$$
$$t = \frac{\lambda}{4n} = \frac{550\,nm}{4(1.38)} = 99.6\ nm$$

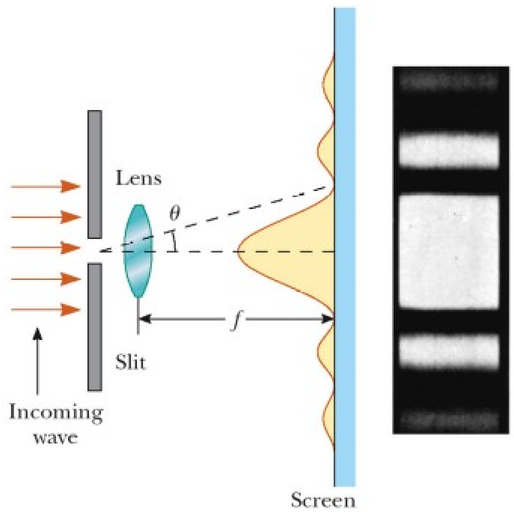
What wavelength(s) in the visible would be most strongly reflected?

$$\lambda = \frac{2nt}{m}$$
$$\lambda_1 = \frac{2nt}{1} = 2(1.38)(99.6\,nm) = 275\ nm$$
$$\lambda_2 = \frac{2nt}{2} = 137\ nm$$
$$etc.$$

These wavelengths are in the ultraviolet, so no visible light will be strongly reflected.

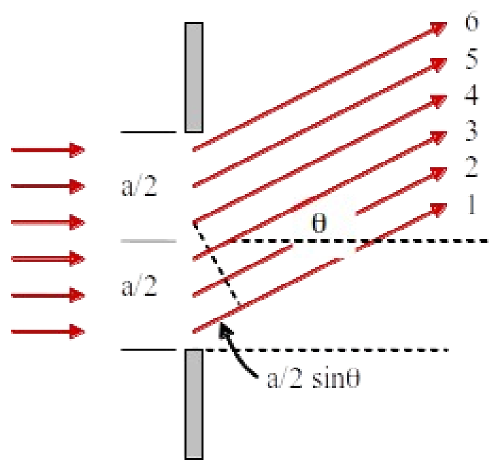
Diffraction

Light that passes through a single opening such as a narrow slit or a pinhole will be spread out. The spread will consist of bright and dark regions, somewhat like the double slit interference effect. Essentially, the light waves emanating from different parts of the slit interfere with one another. This 'self interference' is referred to as *diffraction*. Diffraction also occurs when light shines on the sharp edge of an opaque object and is projected onto a screen. The effect is a bending of the light around the edge giving rise to bright and dark lines near the shadow of the edge of the object on the screen.



The figure shows the diffraction pattern from a single slit. The pattern consists of a central bright region and side bands of lesser intensity.

The positions on the screen where the light intensity is zero correspond to complete destructive interference of the waves from the slit. The position of the first minimum (dark lines) can be obtained from the diagram below. Imagine that the slit is divided into two adjacent parts, each of width *a/2*. The path length difference between waves emanating from points within the slit a distance *a/2* apart (e.g., waves 1 and 4) is *a/2sin θ*. If this path length difference is *λ/2*, then the waves interfere destructively and cancel. If this occurs, then all other wave pairs a distance *a/2* apart (e.g., 2 and 5, and 3 and 6) will also interfere destructively. As a consequence there is complete destructive interference from all the light coming from the slit. The angle at which this occurs is given by



$$\frac{a}{2}\sin\theta = \frac{\lambda}{2} \ ,$$

That is,

$$\sin\theta = \frac{\lambda}{a} \qquad \text{(first diffraction minimum)}$$

Likewise, to find the condition for the second minimum we divide the slit into four parts each with width *a/4*. Using arguments similar to those above, we find

$$\sin\theta = \frac{2\lambda}{a} \qquad \text{(second diffraction minimum)}$$

In general, we find that the positions of the dark stops are given by

$$\sin\theta = \lambda\frac{a}{m},\ m = \pm 1,\ \pm 2,\ \pm 3,\ \ldots \qquad \text{(all diffraction minima)}$$

The maxima are located approximately half-way between the minima.

Using the small angle approximation $\sin\theta \approx \tan\theta = y/L$, the location of the minima is

$$y_m = \frac{\lambda L}{a} m,\ \ m = \pm 1,\ \pm 2,\ \pm 3, \ldots$$

Which give a central bright maximum twice as large than the next order maxima.

Note from this equation how λ needs to be of the same order of a as requested by the condition 2) where now a represents the *size* of the object (see condition 2 for interference to occur).
If $a >> \lambda$ then $\Delta y = y_{m+1} - y_m$ goes to zero and the diffraction patter disappears.

Example

Light of wavelength 640 nm shines through a slit of width a = 0.05 mm and is projected onto a screen 4 m away. What is the angular position of the first minimum?

$$\sin\theta = \frac{\lambda}{a} = \frac{(640e-9m)}{(0.05e-3m)} = 0.0128$$
$$\theta = 0.73^o$$

What is the distance on the screen between the first minima above and below the central maximum (m = 1 and m = -1)?

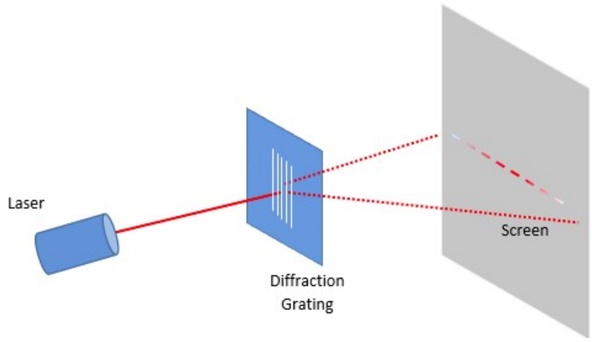
$$\frac{y}{L} = \tan\theta \approx \sin\theta = 0.0108$$
$$y = (0.0108)L = (0.0108)(3m) = 0.0324m = 3.24cm$$
$$\Delta y = 2y = 6.48\ cm$$

Diffraction Grating

A diffraction grating consists of an array of equally spaced slits.

It can be made by scratching groves on a glass plate or sheet of plastic, where the smooth spaces between the scratches act as slits. The grating on the right has N = 600 lines/mm, the spacing between each line is $d = 1/N = 1.66 \times 10^{-6}$ m

When monochromatic light passes through a grating and is projected onto a screen, bright spots (or lines) are seen at positions where the light waves from all slits are in phase.



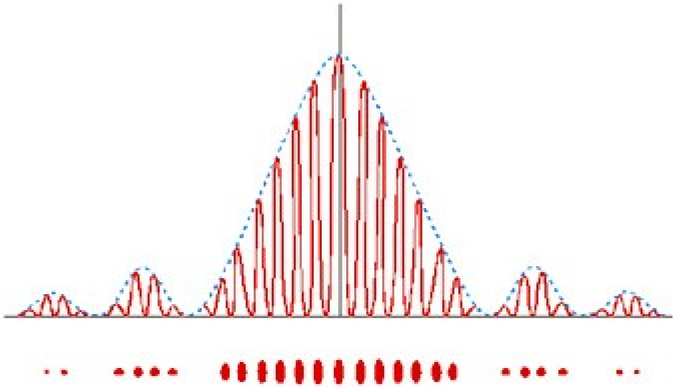
The path length difference between the waves from adjacent slits is an integral number of wavelengths. The conditions for the interference maxima (the bright spots) are the same as for a double slit, that is

$$m\lambda = d\sin\theta,\ m=0, \pm 1, \pm 2, \pm 3, \ldots \qquad \text{(diffraction grating maxima)}$$

where d is the adjacent slit spacing. The difference between a grating consisting of many slits and the double-slit arrangement is that the bright lines for the grating are much sharper. As the number of slits increases, the lines get sharper.

On screen both the pattern of the inference (red line) and the patter of diffraction (blue line) appear.

The maxima given by the equation above refers to the maxima of the interference red line.

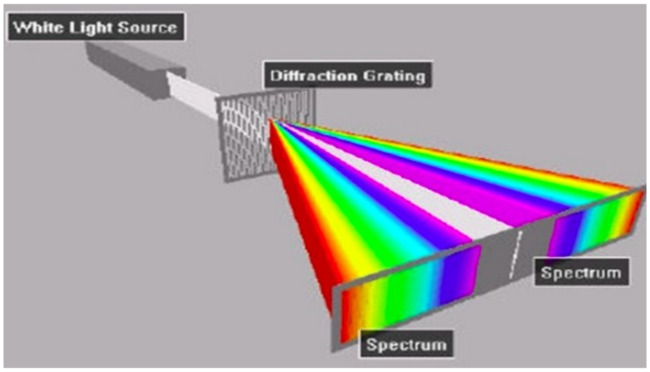


Example

A diffraction grating has 5000 lines per cm. What are the angular positions of the first order and second order diffraction lines for red light with wavelength 640 nm and blue light with wavelength 480 nm?

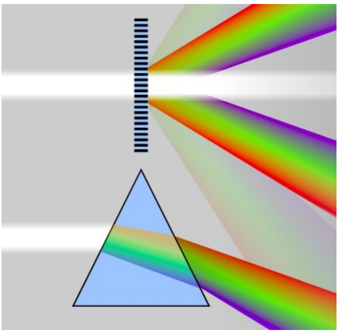
$$d = 1/N = 1/(5000/cm) = 2e-4\ cm$$
$$\sin\theta = \frac{m\lambda}{d}$$
$$\underline{first\ order}\ (m=1)$$
$$\theta_{red} = \sin^{-1}\left(\frac{640e-9\ m}{2e-6\ m}\right) = 18.7^o$$
$$\theta_{green} = \sin^{-1}\left(\frac{480e-9\ m}{2e-6\ m}\right) = 13.9^o$$
$$\underline{second\ order}\ (m=2)$$
$$\theta_{red} = \sin^{-1}\left(\frac{2(640e-9\ m)}{2e-6\ m}\right) = 39.8^o$$
$$\theta_{green} = \sin^{-1}\left(\frac{2(480e-9\ m)}{2e-6\ m}\right) = 28.7^o$$

This example shows how a diffraction grating can be used to disperse light into its component colors, somewhat like a prism.



For a grating, the deviation of the diffracted light from its incoming direction increases with increasing wavelength, whereas for a prism it increases with decreasing wavelength.

Also, with a grating there can be multiple orders, both positive and negative, where the light is dispersed into its component wavelengths, whereas for a prism there is only one region of dispersion.

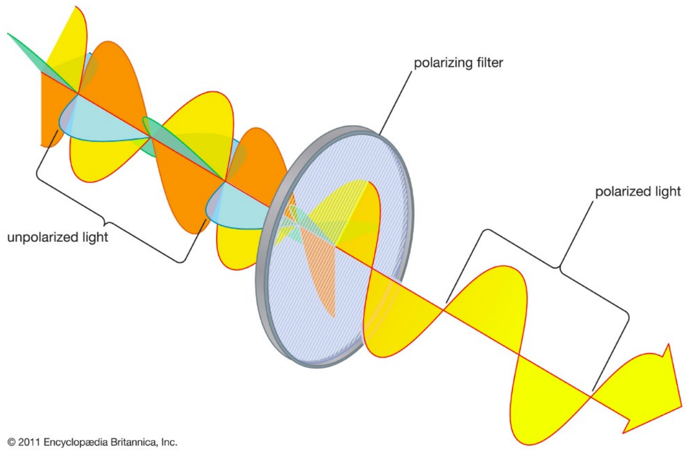


Polarization

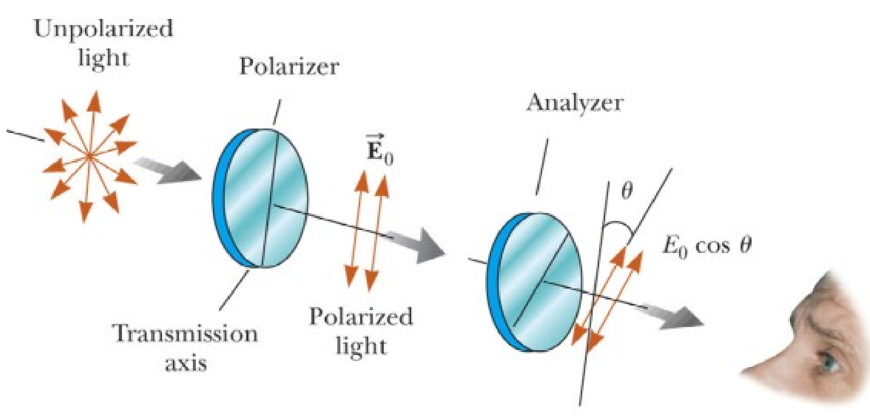
While interference might occur for both longitudinal and transverse waves, polarization only take place for transverse wave as in the case of light where the electric and magnetic fields oscillate perpendicular to the direction of propagation. Natural light from an ordinary source consists of many waves with different directions of oscillation of **E** and **B**. If there is a single, well-defined direction of oscillation then the light is *polarized.*

One way to polarize natural light is to send it through a sheet of Polaroid filter. This is a material that selectively absorbs light that is not polarized in a specific direction.

Before unpolarized light passes thought the filter all possible directions of E (and B) are present. The figure displays three E fields (yellow, orange and blue). For every E there is B field (not displaced) perpendicular to it. After the filter only one direction of oscillation is selected.



A second sheet of Polaroid can then be placed in the path of the polarized light to act as an analyzer. If the polarization axis of the two sheets is parallel, then a maximum amount of light is transmitted through the second sheet. If the polarization axes are perpendicular, then the second sheet absorbs the light so no light is transmitted. This effect is illustrated below.



If E_0 is the amplitude of the electric field of the polarized light before the analyzer, then the amplitude after the analyzer is $E_0 cos\ \theta$. Since the intensity of light is proportional to the square of the amplitude, then the final intensity is given by

$$I = I_0 \cos^2\theta$$

This equation is called *Malus' law*.

Example

Light from a light bulb has intensity I_0, calculate the intensity I after it passes through a polarized filter.

Since natural light has all possible values of the angle θ, Malus law applies to the average value of $cos^2(\theta)$

$$<\cos^2(\theta)> = \frac{1}{2\pi}\int_0^{2\pi} \cos^2(\theta)\, d\theta = \frac{1}{2}$$

and so

$$I = \frac{1}{2} I_0$$

Example

Unpolarized light I_0 passes through three sheets of Polaroid in succession. The orientation of the polarization axis of the sheets is 0°, 30°, and 90°. If the intensity of light after the first sheet is I_1, what is the intensity after the second sheet?

$$I_2 = I_1 \cos^2(30) = 0.75\, I_1$$

After the third sheet? The angle between the third and second sheets is 90° – 30° = 60°.

$$I_3 = I_2 \cos^2(60) = 0.75 I_1 \cos^2(60) = 0.1875\, I_1$$

How is I_3 compared to I_0?

$$I_3 = 0.1875(1/2\, I_0) = 0.0937\, I_0$$

Note that light passes through the last polarizer even though its polarization axis is perpendicular to that of the first polarizer. Without the intermediate polarizer no light would pass through the last one.

Polarization by reflection

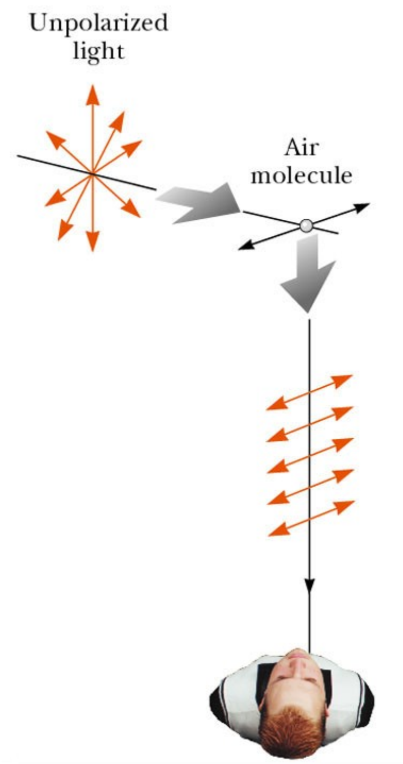
When unpolarized light strikes a surface, both the reflected and refracted light waves are partially polarized. At a specific angle, called the *Brewster angle*, the reflected light is completely polarized. This occurs when the angle between the reflected and refracted light is 90^{o}.

Brewster's angle is given by

$$\tan\theta_B = \frac{n_2}{n_1}$$

In this equation n_1 is the index of refraction of the medium from which the light originates and n_2 is the index of refraction of the medium into which some of the light is refracted.

Polarization by scattering



When light that is incident upon a system of particles, such as the molecules in a gas, the particles absorb and re-radiate some of the light. The oscillating electric field associated with the incoming light causes the charges in the molecules to oscillate in the same direction and re-radiate light with the same polarization. This means that if the incident light is unpolarized, then the light that is scattered at 90^{o} must be polarized since the associated electric field must be perpendicular to the direction of propagation. This explains why sunlight that is scattered from the earth's atmosphere is polarized.

Optical activity

Some transparent materials will rotate the direction of polarization of polarized light. The degree of rotation will depend on the thickness of the material and the wavelength of light. Such materials are termed 'optically active'. Essentially, this is because the molecules of the material are asymmetric. The index of refraction of these materials depends on the direction of polarization of the light as it passes through the material. If polarized white light that passes through an optically active is viewed through a sheet of Polaroid, then a colored pattern will be observed that depends on the orientation of the Polaroid sheet. This pattern can also vary with stress in the material, since stress can also affect the optical activity.