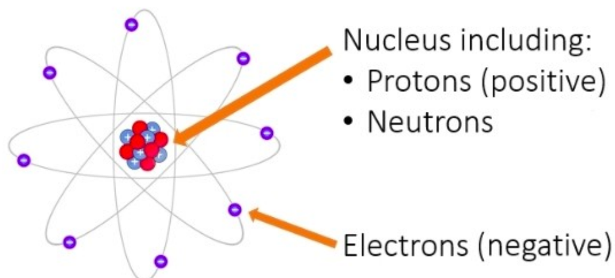
Electric Field

Electric Charge

<u>Atomic structure</u>

All materials can have a charge, which we refer to as either “positive” or “negative”. The origin of this charge is to be understood within the proprieties of the elementary particles which make an atom.

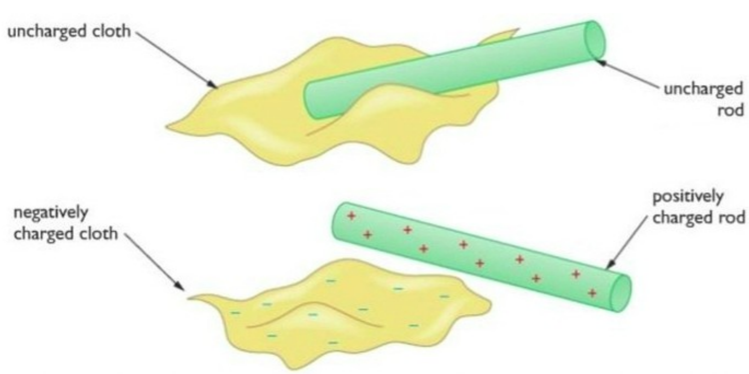
The atom consists of a positively charged nucleus surrounded by negatively charged electrons. The nucleus consists of protons, which have positive charge, and neutrons, which have no charge.



The charge of a proton is the same in magnitude but opposite in sign to that of an electron. In a neutral atom there are an equal number of electrons and protons and its net charge is zero. The overall net charge of the material composed of neutral atoms is zero as well.

Charging a material

When two dissimilar materials (e.g., a plastic rod and cloth, a glass road and a cloth, or a comb and your hair) are rubbed together, then electrons can transfer from one material to the other so that the material with an excess of electrons has a net negative charge and the material with a deficit of electrons has a net positive charge. This will occur because the two materials have a different affinity for electrons.



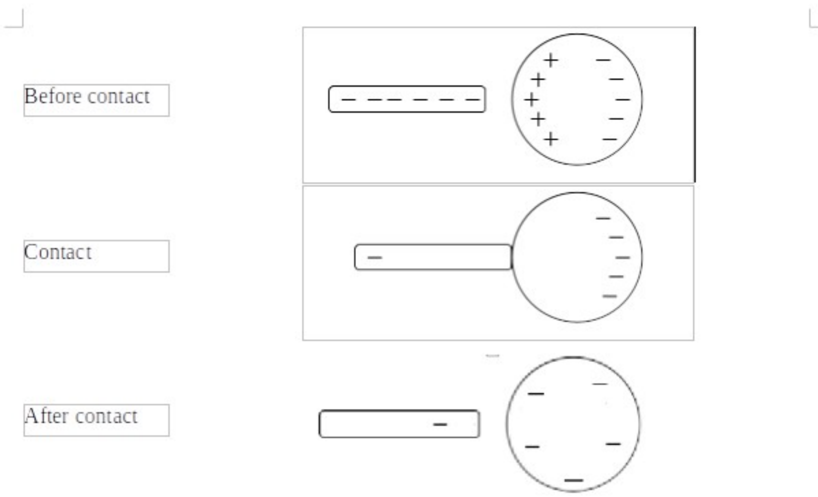
In the picture above a glass rod is charged with a cloth. The rod becomes positively charged because some of its electrons are taken away from the cloth which become negatively charged.
If the rod were a plastic rod it would take the electrons away from the cloth and the plastic rod becomes negatively charge.

Materials can be electrically classified by how well charges move through the materials. In a *conductor*, charges flow freely. Examples of conductors are copper, silver, gold, and aluminum. In an *insulator*, the flow of charges is virtually zero. Examples of insulators are glass, rubber, and wood. In a *semiconductor*, charges can flow weakly only in one direction. Examples of semiconductors are silicon, germanium, and gallium arsenide. Later we will see that the degree to which charges can flow in a material can be quantitatively characterized by its *conductivity*.

Forces exist between charges. Like charges repel (e.g., positive-positive or negative-negative), while unlike charges attract (positive-negative).

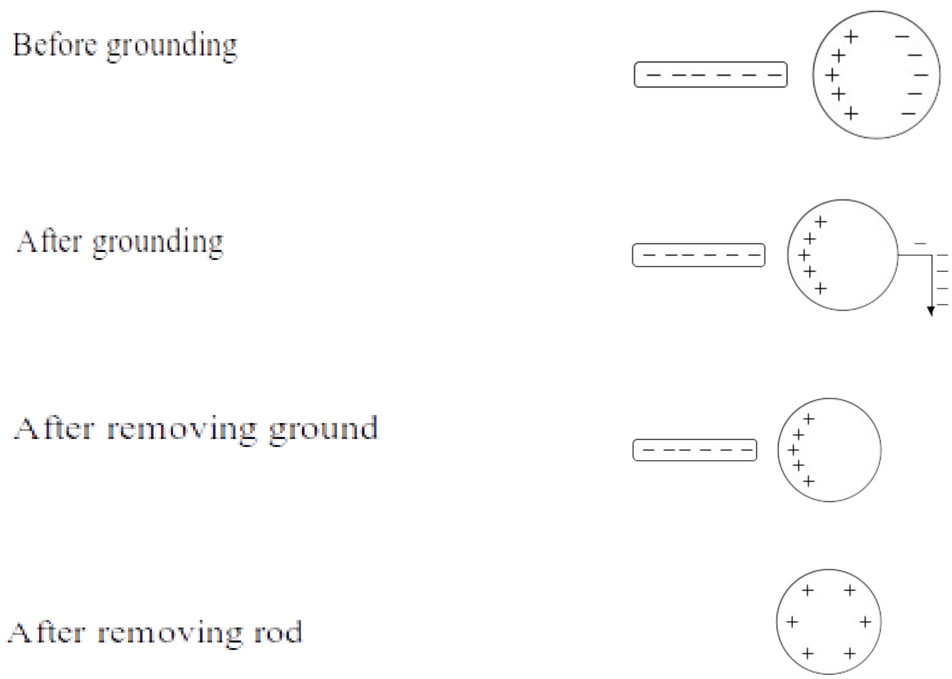
<u>Conduction</u>

Some of the charge on a negatively (or positive) charged rubber rod can be transferred to an initially uncharged isolated metal sphere by touching the rod to the sphere, as shown in the figure below. The rod initially pushes electrons to the opposite side of the sphere, making one side positively charged and the other side negatively charged. When the rod touches the sphere, electrons transfer from the rod to the sphere because of the attraction of opposite charges. When the rod is removed, then the excess electrons remain on the sphere and become uniformly distributed over the surface because of their mutual repulsion.



Induction

A positive charge can be induced onto the metal sphere without bringing the negatively charged rod into contact with the sphere. Instead, a conducting wire connects the sphere to a ‘ground’ (e.g., a copper pipe buried in the earth). This ‘ground’ provides an infinite reservoir of electrons to flow to or from the conducting sphere.



Electric charge is discrete.

The smallest unit of charge is that of the electron or proton. The magnitude of the electronic charge is

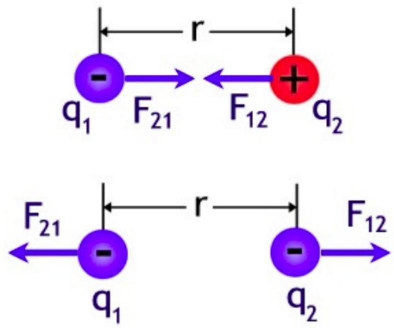
$$e = 1.60\, x 10^{-19}\, C$$

The charge of the proton is $q_p = e$ and the charge of the electron is $q_e = -e$. All charges are integral multiples of e. If a macroscopic object (as a plastic rod) has a net charge Q then Q can only assume the value $Q = ne$ where n is an integer. For example an object with a net charge $Q = 2.17 \text{ x } 10^{-19}$ C does not exists in nature.

If a material has a net charge, then it can be used to charge a conductor either by direct contact (conduction) or indirectly without touching (induction).

Coulomb's Law

Two point charges located at some distance *r* away from each others experience a attractive or a repulsive force depending if their sign of charge is opposite or equal.



The magnitude of the force between the two point charges is given by Coulomb's law:

$$F = k_e \frac{q_1 q_2}{r^2}$$

q_1 and q_2 are the charges and *r* is the distance between the charges. k_e is the Coulomb constant and is given in the SI system by

$$k_e = 8.99 x 10^9 N \cdot m^2 / C^2$$

The SI unit for charge is the coulomb (C).
In Coulomb's law *F* is positive and repulsive if q_1 and q_2 have the same sign and is negative and attractive if q_1 and q_2 have the opposite signs.

Coulomb's law is mathematically very similar to the universal law of gravitation between two point masses, which is

$$F = G \frac{m_1 m_2}{r^2}$$

where *G* is the universal gravitation constant. Both forces are proportional to the product of the charges or masses and inversely proportional to their separation. The gravitational force, however, is always attractive (there are no negative masses); whereas, the electrical force can be attractive or repulsive.

Example

Compare the magnitude of the electrical and gravitational forces between the electron and proton in the hydrogen atom.

Given: $m_e = 9.11 \times 10^{-31}$ kg, $m_p = 1.67 \times 10^{-27}$ kg, $r = 5.3 \times 10^{-11}$.

$$F_e = k_e \frac{|q_e q_p|}{r^2} = (8.99\,x\,10^9\,N \cdot m^2/C^2)\frac{(1.6\,x\,10^{-19}C)^2}{(5.3\,x\,10^{-11}m)^2} = 8.19\,x\,10^{-8}\,N$$

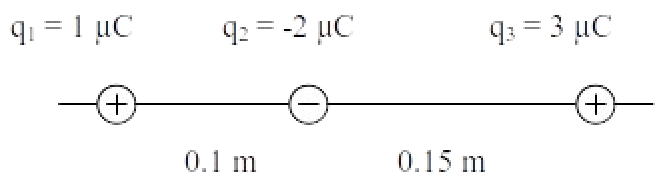
$$F_g = G\frac{m_p m_e}{r^2} = (6.67\,x\,10^{-11}\,N \cdot m^2/kg^2)\frac{(1.67\,x\,10^{-27}\,kg)(9.11\,x\,10^{-31}kg)}{(5.3\,x\,10^{-11}m)^2} = 3.61\,x\,10^{-47}\,N$$

$$\frac{F_e}{F_g} = \frac{8.19\,x\,10^{-8}\,N}{3.61\,x\,10^{-47}\,N} = \underline{2.27\,x\,10^{39}}$$

The electrical forces in an atom are so large compared to the gravitational forces that the gravitational forces can be completely neglected. When considering ordinary masses, gravity is much more important since the net charge on the masses is relatively small.

Example

Find the force on the charge q_2 in the diagram below due to the charges q_1 and q_2.



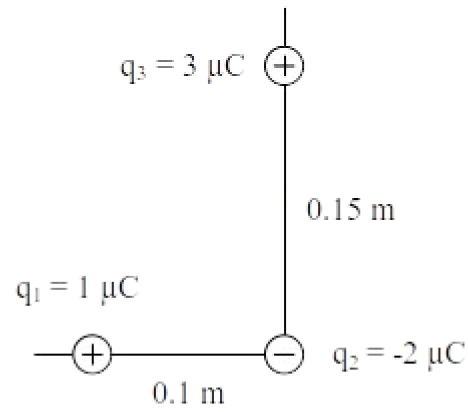
$$F_{12} = k_e \frac{|q_1||q_2|}{r_{12}^2} = (8.99\,x\,10^9\,N \cdot m^2/C^2)\frac{(1\,x\,10^{-6}C)(2\,x\,10^{-6}C)}{(0.1\,m)^2} = 1.8\ N\ (to\ the\ left)$$

$$F_{32} = k_e \frac{|q_3||q_2|}{r_{32}^2} = (8.99\,x\,10^9\,N \cdot m^2/C^2)\frac{(3\,x\,10^{-6}C)(2\,x\,10^{-6}C)}{(0.15\,m)^2} = 2.4\ N\ (to\ the\ right)$$

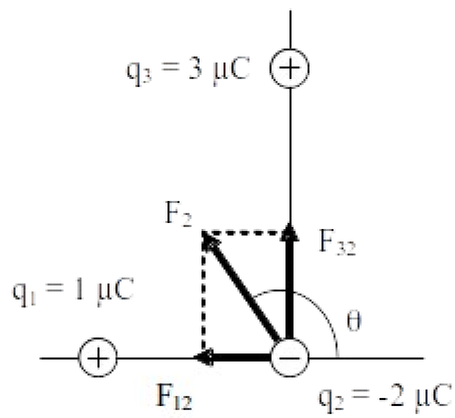
$$F_2 = -F_{12} + F_{32} = -1.8\ N + 2.4\ N = 0.6\ N\ (to\ the\ right)$$

Example

Find the force on q_2 in the diagram to the right.

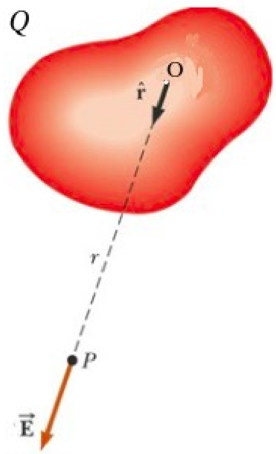


$$F_x = -F_{12} = -1.8\ N$$
$$F_y = F_{32} = 2.4\ N$$
$$F_2 = \sqrt{F_x^2 + F_y^2} = \sqrt{(1.8)^2 + (2.4)^2} = 3.0\ N$$
$$\tan\theta = \frac{F_y}{F_x} = \frac{2.4}{-1.8} = -1.33$$
$$\Rightarrow \theta = 126.9^o\ (ccw\ \ from\ \ pos.\ \ x-axis)$$



Electric Field

The electric field is the fundamental concept for electric phenomena. It also helps to visualize the fact that charges can exert forces on each other without being in contact. A total electric charge Q present on an object produces an electric field **E** in the space surrounding it, analogous to the gravitational field due to a mass.



The charge Q is distributed on the red object above. The electric field at P is shown

In order to determine **E** due to Q and at P we need a second charge, called the test charge. If q satisfies:

1- It's positive
2- If its numerical value is very small compared to Q in such a way that E_q, the electric field produce by the test charge can be ignored.

then q is called a test charge $q = q_0$.

To find **E** due to Q at a location P, the test charge q_0 is place at P to 'test' the Coulomb force F between the Q and q_0 . The the electric field of Q is evaluate by dividing out the test charge.

$$E = \frac{F}{q_0}$$ (units are N/C)

The direction of **E** is the same as the direction of **F** on the test charge.

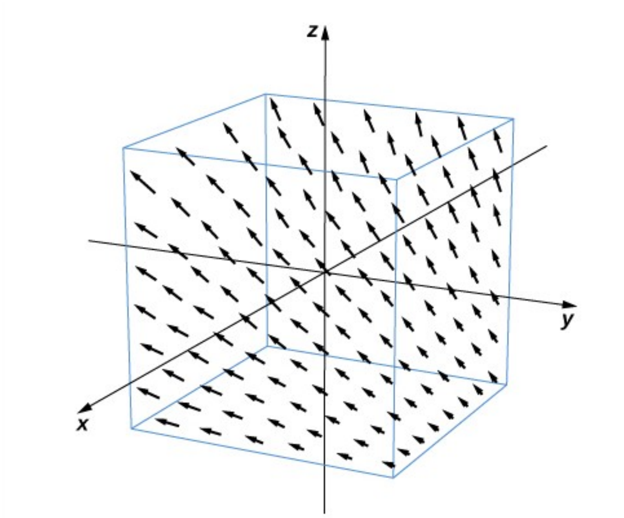
If we know **E** due to some charge Q, we can find the force on any other charge q from

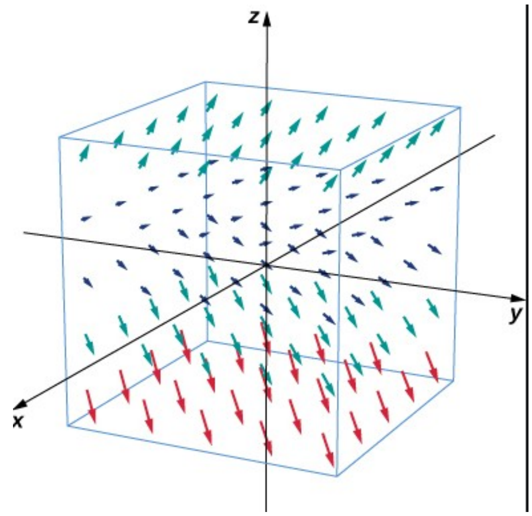
$$F = qE$$

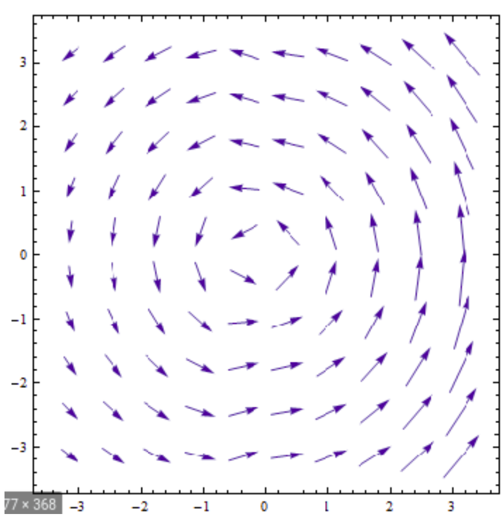
Here q is any charge, not necessarily q_0.

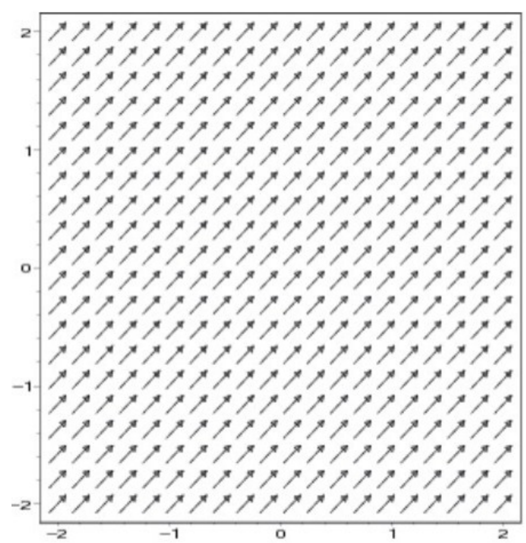
<u>The Electric field is a vector field</u>

There is a vector at each point in space.









Electric field due to a point charge

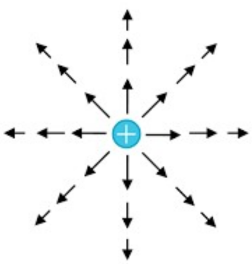
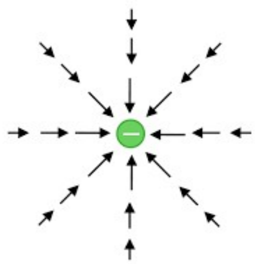
The magnitude of the force on a test charge q_0 due to a point charge Q is

$$F = k_e \frac{|Q||q_0|}{r^2} \; .$$

Thus, the magnitude of the electric field due to Q is

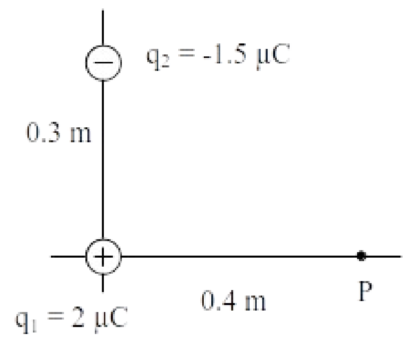
$$E = \frac{F}{q_0} = k_e \frac{|Q|}{r^2} \; .$$

Notice how the magnitude of **E** depends on Q only and not q_0. Every charge is a source of its own electric field in the surrounding space. The direction of **E** is radially away from positive charges and radially toward negative charges.

Example

Find the electric field at P due to charges q_1 and q_2.



$$E_1 = k_e \frac{|q_1|}{r_1^2} = (8.99 x 10^9) \frac{(2 x 10^{-6})}{(0.4)^2} = 1.13 x 10^5 \; N/C$$

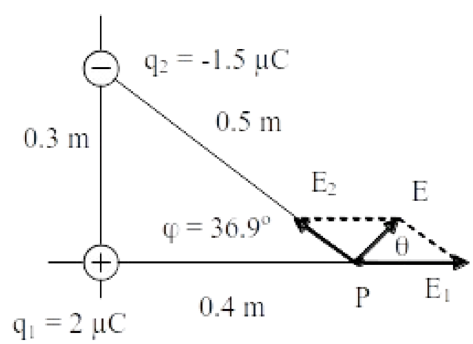
$$E_2 = k_e \frac{|q_2|}{r_2^2} = (8.99 x 10^9) \frac{(1.5 x 10^{-6})}{(0.5)^2} = 5.4 x 10^4 \; N/C$$

$$E_x = E_1 - E_2 \cos\phi = 1.13 x 10^5 - 5.4 x 10^4 \cos(36.9)$$
$$= 6.9 x 10^4 \; N/C$$

$$E_y = E_2 \sin\phi = 5.4 x 10^4 \sin(36.9) = 3.24 x 10^4 \; N/C$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(6.9 x 10^4)^2 + (3.24 x 10^4)^2}$$
$$= 7.6 x 10^4 \; N/C$$

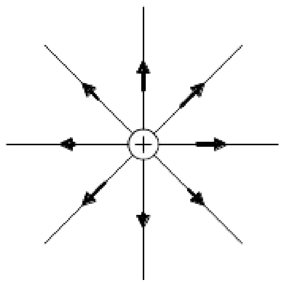
$$\theta = \tan^{-1}(E_y / E_x) = \tan^{-1}(3.24/6.9) = 25^o$$



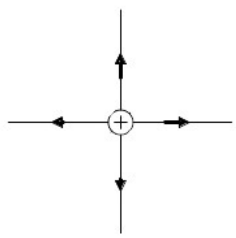
Electric Field Lines

The strength and direction of the electric field can be graphically displayed using *electric field lines*. These are lines that originate on positive charges and terminate on negative charges. The number of lines originating or terminating on a charge is proportional to the magnitude of the charge. The direction of the electric field is the direction of the tangent to a line and the strength of the electric field is proportional to the density of lines.

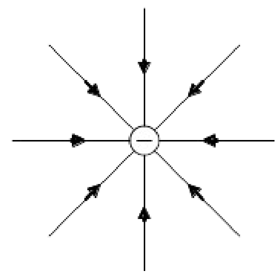
The field lines for a positive point charge and a negative point charge are shown below.



positive charge Q_1



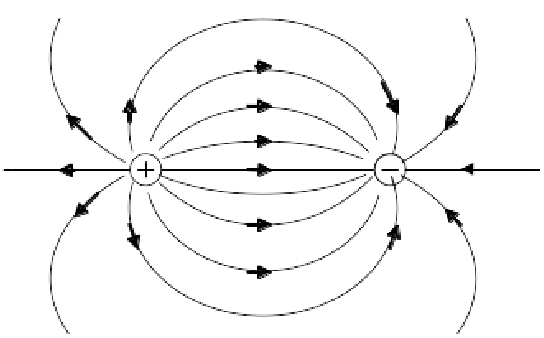
positive charge Q_2

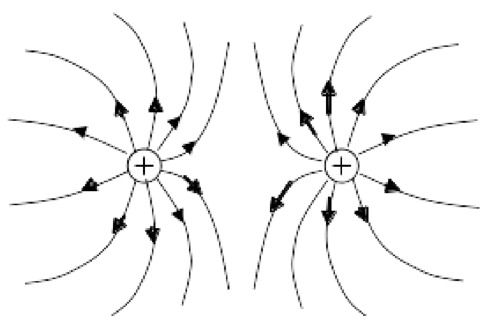


negative charge Q_3

$$Q_1 = -Q_3 > Q_2$$

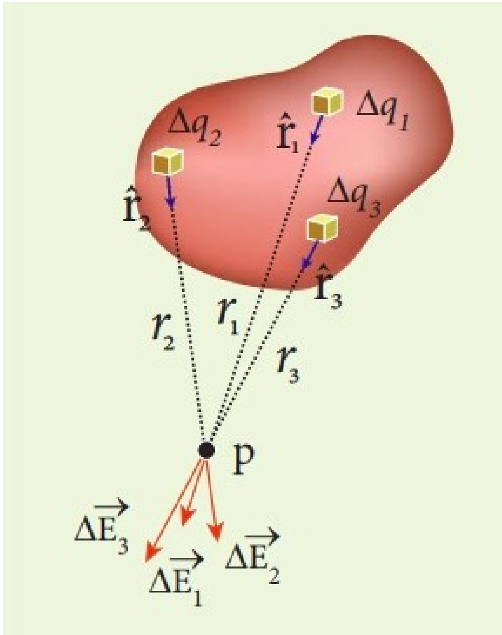
If a positive and a negative charge are brought close together, then the field lines are a superposition of the field lines for the separate charges.
Likewise, for two equal positive charges which are close together:





Electric field due to a continuous charge distribution

For a continuous distribution of charge, the electric field can be found by summing up all the contributions due to the elements of the distribution.



$$\Delta\vec{E} = k_e \frac{\Delta q}{r^2}\hat{r} \ ,$$

where Δq is an elemental part of the charge and $\hat{r}$ is a unit vector pointing away from the charge. Summing, we have

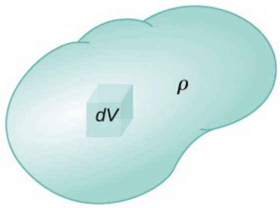
$$E \approx \sum \Delta E = k_e \sum \frac{\Delta q}{r^2}\hat{r}$$

In the limit of infinitesimal charge elements $\Delta q \Rightarrow dq$

$$\vec{E} = k_e \int \frac{dq}{r^2}\hat{r}$$

Charge densities

Volume charge density $\rho = \frac{dq}{dV} \Rightarrow dq = \rho dV$

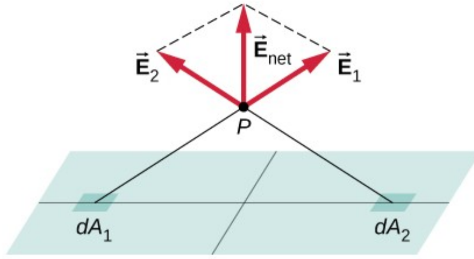


Surface charge density $\sigma = \frac{dq}{dA} \Rightarrow dq = \sigma dA$

Linear charge density $\lambda = \frac{dq}{dL} \Rightarrow dq = \lambda dL$

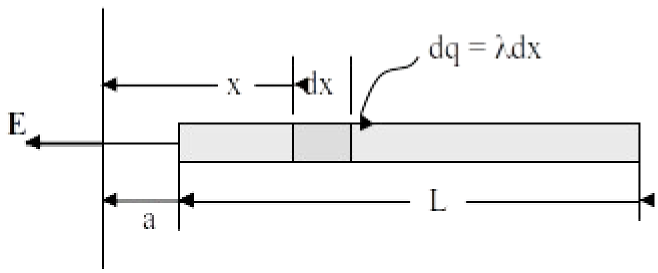
Example

$$\vec{E} = k_e \int \frac{dq}{r^2}\hat{r} = k_e \int \frac{\sigma}{r^2}\hat{r}\, dA$$



Electric field due to a uniform charged rod

Electric field near the end of a uniformly charged rod.



A piece of the rod of length dx has charge dq = λdx. So. The rod has charge *Q*, length *L*, and charge per unit length $\lambda = dq/dL = Q/L$.

$$dE = k_e \frac{dq}{x^2} = k_e \frac{\lambda\, dx}{x^2}$$

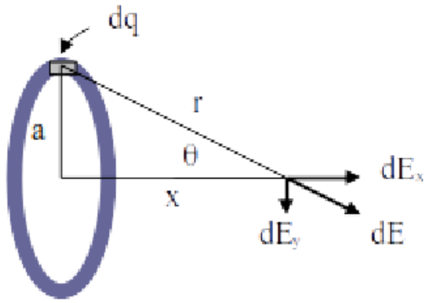
$$E = \int_a^{L+a} k_e \frac{\lambda\, dx}{x^2} = -k_e \lambda \left[\frac{1}{x}\right]_a^{L+a} = k_e \lambda \left(\frac{1}{a} - \frac{1}{L+a}\right) = k_e \lambda \frac{L}{a(L+a)}$$

$$E(a) = k_e \frac{Q}{a(L+a)}$$

Note that this expression for *E* is not the same as for a point charge. However, in the limit that $L \to 0$, it approaches the expression for a point charge. (Show that this is true.)

Uniform charged ring

We can easily calculate the electric field on the axis of a uniform ring of charge.



$$dq=\lambda\, dl=\lambda 2\pi da$$

$$Q=\int dq=\int \lambda 2\pi da=2\pi\lambda a$$

$$\lambda=\frac{Q}{2\pi a}$$

The electric field due to *dq* along the *x* axis, at a distance *x* from the center has magnitude

$$dE=k_e\frac{dq}{r^2}$$

Considering the contributions from all the charge elements, the *y*-components add to zero and the net field is along the *x*-direction with magnitude

$$E=\int dE_x=\int\cos\theta\, dE=k_e\int\cos\theta\frac{dq}{r^2}=k_e\int\frac{x}{r}\frac{dq}{r^2}=k_e\frac{x}{r^3}\int dq=k_e\frac{Q\ x}{r^3}$$

$$E=k_e\frac{Q\ x}{\left(x^2+a^2\right)^{3/2}}$$

At a fixed value of x the infinitesimal *dq* along the ring contribute equally to the E. The integration over the charge does not depend on x and r which are taken out of the integral. What is the limiting expression for $x >> a$? (given by lim $a \rightarrow 0$)

$$E=k_e\frac{Q}{x^2}$$ point charge

How about $x = 0$?

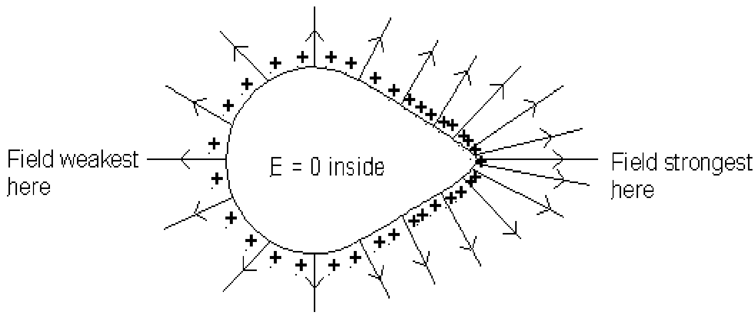
$$E = 0$$ no electric field

This indicates there is a location between $x = 0$ and $x = \infty$ where E has a max value. To find the location we solve $dE(x)/dx = 0$ which gives $x=\pm\left(1/\sqrt{2}\right)a$.

Conductors in Equilibrium

Consider an isolated conductor, such as copper, silver or gold, in which the charges are in equilibrium. (No batteries, for example, to drive a current through the conductor.) The conductor will have the following properties:

1. **E = 0** everywhere inside the conductor.
2. Any excess charge will reside on the surface.
3. **E** just outside the conductor will be perpendicular to the surface.
4. Charge concentrates more on the surface regions which have the greater curvature.



We consider each property separately.

1. **E** (inside) = 0. Conductors have electrons that can freely move under the influence of an electric field. If **E** (inside) were not zero, then there would be currents inside. However, we are considering the properties of conductors in electrostatic equilibrium.

2. Excess charge resides on surface(s). Since like charges repel, if there were excess charge inside, then the repulsive forces on these charges would push them as far apart from each other as possible, which would mean to the surface. The binding force of the electron to the metal would keep them at the surface (unless the charge was so great that the electrostatic repulsion would overcome the binding force and the charge would 'arc' to another object.)

3. If **E** is perpendicular to the surface, then **E** (parallel) = 0. If **E** (parallel) were not zero, then there would surface currents. Again, we are assuming that the charges are in equilibrium.

4. The curvature reduces the component of the repulsive force between charges that is tangent to the surface, which allows the charges to be closer together. Because the electric field is greater near sharp points, these are the places where the charge is most likely to arc if the charge on the conductor becomes too great. Consequently, the electric field is greater at these sharp regions.

Motion of a charge in a external electric field

Example

A point charge q_1 = 3.8 x 10^{-10} C with mass m_1 = 6.85 x10^{-11} kg is placed one meter away from another point charge q_2 = 4.11 x 10^{-10}. Find the initial acceleration of q_1.

The electric field due to q_2 is $E_2 = k\frac{q_2}{r^2} = 3.70\frac{N}{C}$. To find the acceleration of q_1:

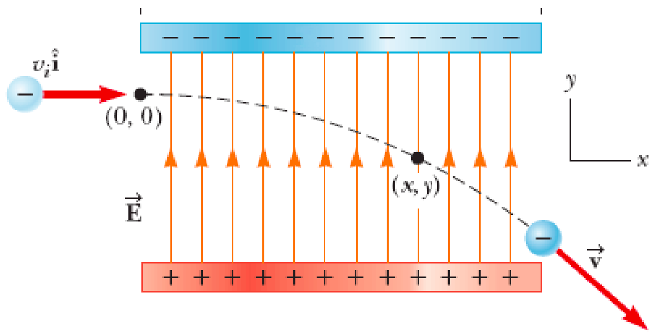
$$F = m_1 a \quad \Rightarrow \quad q_1 E_2 = m_1 a$$
$$a = \frac{1}{m_1} q_1 E_2 = 20.52 \quad m/s^2$$

Example

An electron is released from rest in a uniform electric field E = 1000 N/C. How long does it take the electron to travel a distance of 2 cm?

In a uniform electric field, a charge undergoes an acceleration given by

$$a = \frac{F}{m} = \frac{qE}{m} \qquad \text{since } E \text{ is uniform } a \text{ is constant.}$$



Using the kinematic equations for constant acceleration

$$x = \tfrac{1}{2}at^2,$$
$$a = \frac{qE}{m} = \frac{(1.6\,x\,10^{-19}C)(1000\,N/C)}{9.11\,x\,10^{-31}kg} = 1.76\,x\,10^{14}\,m/s^2$$
$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(0.02\,m)}{1.76\,x\,10^{14}\,m/s^2}} = 1.51\,x\,10^{-8}\,s$$

How fast is the electron going after 2 cm?

$$v = at = (1.76\,x\,10^{14}\,m/s^2)(1.51\,x\,10^{-8}\,s) = 2.66\,x\,10^6\,m/s$$