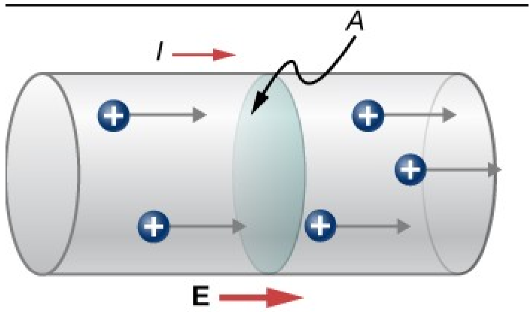
Current and Resistance

The electric current *I*

Current is the flow of charge. Specifically, it is the charge per unit time that flows through a given cross-sectional area *A*. (e.g., the cross section of a wire). The current at a given time, or instant current *I*(*t*) is

$$I(t) = \frac{dq}{dt}$$ (unit = C/s = ampere (A))



Where *dq* is the infinitesimal charge. In electrostatics, the electric field inside the conductor is zero, the difference in the values of the potential at the ends of the conductor ΔV is zero, and the charges are at rest. If a metal carries a current is because of a non-zero electric field present within the wire which is generated by a potential difference ΔV. This could happen, for example, by connecting the ends of the conductor to the terminals of a battery.
The total charge passing thought the cross section in a time interval is:

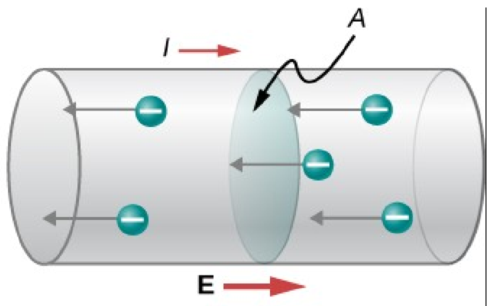
$$\Delta Q = \int dq = \int I(t)\,dt$$

The current has a direction. The accepted convention for the direction of current flow is the direction of the flow of positive charges *dq*.

In metal, however, the current consists of a flow of electrons $I_{electrons}$ so that the physical current of the electrons has the opposite direction of *I*(*t*)

$$I_{electrons} = -I$$

and same magnitude.

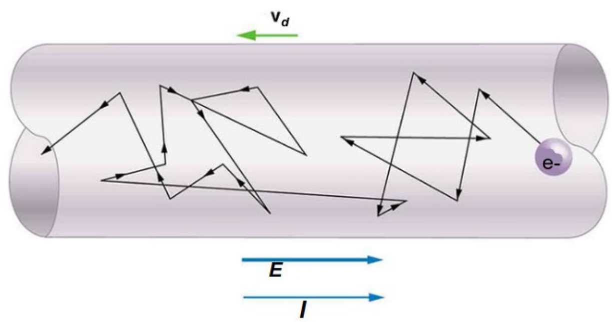


For practical reasons (at least at 106 level) this distinction is not necessary and we will pretend that the current inside a metal is describe as the flow of positive charges from the higher to the lower potential, i.e. following the direction of the electric field.

Microscopic interpretation

In a conductor the electrons travel in zig-zag paths as they collide with atoms. The average distance between collisions is typically tens of nanometers. If there is no potential difference, then the movement of the electrons is random and there is no net current.

If a potential difference is applied the electron slowly drift as they continue to move in a zig-zag path.



The current in the conductor can be expressed in terms of the number of conducting electrons per unit volume n and their drift speed v_d: since there is a minimum value of the electric charge e, the infinitesimal charge can be written as $dq = e\,dN$ where N is the number of electrons. The electrons move over a distance dx and therefore through an infinitesimal volumes $dV = Adx$. The magnitude of the electron current is

$$I = \frac{dq}{dt} = \frac{edN}{dt}\frac{dV}{dV} = n\,e\,A\,\frac{dx}{dt}$$

where $n = dN/dV$ is the number density. The term $v_d = dx/dt$ is referred as the *drift speed:* it provides an estimate of the average velocity of the electrons along the wire, in the opposite direction of the electric field. Thus,

$$I = n\,e\,A\,v_d$$

and

$$v_d = \frac{I}{n\,e\,A}$$

Example

A 16-gauge copper wire had a diameter of 1.29 mm and is rated to carry a maximum current of 22 A. What would be the drift speed of the electrons if the current were 10 A?

In order to find v_d from the above formula, we must first find n, the number of conduction electrons per unit volume (*or* number density).

Mass density of copper, ρ_{Cu} = 8.96 g/cm^3
Atomic mass of copper, M_A (= A) = 63.55 g/mole
Avogadro Number N_A = 6.023 x 10^{-23} mole^{-1}

The number of moles per cm^3 is ρ_{Cu} / M_A (check units) and the number of atoms per cm^3 is n = $(N_A \rho_{Cu})/M_A$. There is one conduction electron per atom of copper, so

$$n = \frac{N_A \rho_{Cu}}{M_A} = \frac{(6.023 x 10^{23} / mole)(8.96 g/cm^3)}{(63.55 g/mole)} = 8.49 x 10^{22} / cm^3 = 8.49 x 10^{28} / m^3$$

Then

$$v_d = \frac{I}{neA} = \frac{I}{nq\pi (d/2)^2} = \frac{(10 A)}{(8.49 x 108/m^3)(1.6 x 10^{-19} C)\pi (1.29 x 10^{-3} m/2)^2}$$
$$= 5.63 x 10^{-4} m/s = 0.563 mm/s$$

The drift speed is very small compared with the average speed of the electrons between collisions, which is of the order of 10^6 m/s. At the rate of 0.563 mm/s it would take electrons nearly 1 hr to travel the length of a 2 m wire.

If we turn on a lamp, then why does the light come on so quickly? The reason is that the electric field propagates through the wire at the speed of light *c* (= 300,000 km/s) and quickly begins to push electrons along all parts of the wire and through the filament of the light. This would be analogous to turning on the water to a hose. If the hose is full of water, then when you turn on the spigot the water almost instantaneously comes out the end, whereas if the hose is empty then you have to wait a bit for the water to emerge from the end. If the hose if full of water, then the water comes out quickly because the pressure increase is transmitted rapidly through the hose. The pressure difference in the hose is analogous to the potential difference in the wire.

Ohm's Law

The resistance of a conductor is operationally (= by taking measurements of dynamical quantities such I and ΔV) defined as

$$R = \frac{\Delta V}{I}$$ unit = V/A = ohm (Ω)

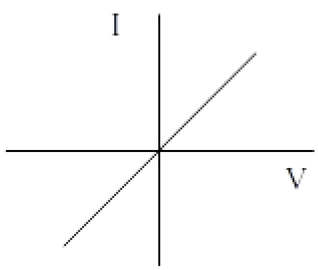
This is often written in the form

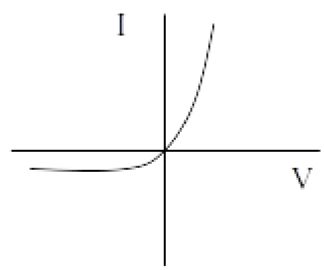
$$\Delta V = IR \quad \text{or} \quad I = \frac{1}{R} \Delta V$$

The unit for resistance is the Ohm, symbol Ω where 1 Ohm = 1 V/ 1 A.
In metals the voltage is proportional to the current over a wide range of current and voltage, and R is constant. In this case the equation above is referred to as 'Ohm's law'. In some materials or devices the voltage is not proportional to the current and the

material or device is said to be non-ohmic. The equation above still applies, but *R* depends on the values of *I* and Δ*V*.
Note: In circuits we often write the potential difference Δ*V* between two points as just *V* since the lowest value of the potential is set to zero, so that we might write *V* = *IR*.

In an ohmic device (constant *R*) the *I-V* curve would be linear, and in a non-ohmic device the *I-V* curve would be nonlinear. An example of a non-ohmic device would be a *diode*, in which the current flows more easily in one direction than the other. In other words, in one direction the resistance is low and in the reverse direction the resistance is high.





Fluid Analogy

The flow of current in a conductor is analogous to the flow of a fluid in a pipe. Current is analogous to flow rate (kg/s or m^3/s). Potential difference is analogous to pressure difference between two points in the pipe, which drives the flow. Electrical resistance is analogous to the resistance to the flow of the fluid in the pipe. The resistance could be affected by the diameter and length of the pipe and by the intrinsic causes of the resistance (viscous effects, obstructions) in the pipe.

Conductivity and Resistivity

The current density *J* is defined as the current per unit cross-sectional area of a conductor.

$$J = \frac{dI}{dA}$$

If the current density is constant then *J* = *I/A* and

$$J = nev_d$$

As discussed earlier, as the electrons travel through a conductor under the influence of an applied force = *eE*, they collide with atoms and impurities and accelerate between collisions in a direction opposite to the electric field. The drift velocity is the average velocity they acquire during this acceleration, which is given by

$$v_d = a\tau = \frac{eE}{m}\tau ,$$

where τ is the average time between collisions. The current density becomes

$$J = nev_d = \frac{ne^2 E}{m}\tau .$$

This is equivalent to writing

$$J = \sigma E ,$$ where

$$\sigma = \frac{ne^2 \tau}{m}$$

is the *conductivity* of a given material. It is refereed as conductivity since for a fixed *E* greater values of *s* correspond to higher values of *J*. Different materials have different values of *s* which depends on its intrinsic proprieties such as *n* and τ. For example silver has higher conductivity than platinum which indicates that silver is a better conductor.

The resistivity of a material indicates how difficult is for the charge to flow through it. As such is given by

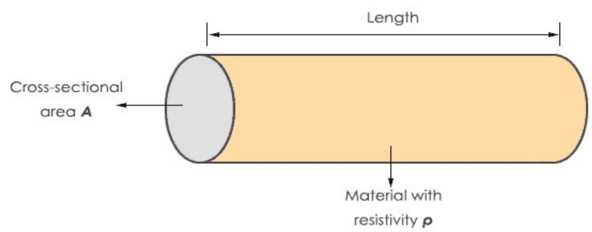
$$\rho = 1/\sigma = \frac{m}{ne^2 \tau}$$

The r*esistivity* ρ (not to be confused with mass density) is a measure of the intrinsic contribution of a material to its resistance and is independent of the geometry of the material. Good conductors, such as copper or silver, have low resistivity, whereas poor conductors such as carbon have high resistivity.

The *resistance R* of a material depends on its resistivity and also on its shape.

For a wire with a uniform cross-sectional area *A* the resistance is proportional to the length *l* and inversely proportional to *A* (unless the area is *extremely* small).

$$R = \rho \frac{l}{A}$$



The unit for the resistance R defines the unit for resistivity to be Ω-m.

Example

What is the resistance of a copper wire that has a length of 10 m and a diameter of 2 mm?

The resistivity of copper at room temperature is 1.7 x 10^{-8} -m. Then

$$R = \rho \frac{l}{A} = (1.7\, x\, 10^{-8}\, \Omega \cdot m) \frac{(10\, m)}{\pi\, (10^{-3}\, m)^2} = 0.054\ \Omega$$

This is a small resistance compared with the resistance of typical circuit elements, which means that in typical circuits with copper wires the resistance of the wires can be neglected.

Temperature Dependence of Resistance

The resistivity and resistance of a substance varies with temperature. If the temperature change is not too great, then the fractional change in resistance is approximately proportional to the temperature change.

$$\frac{\Delta R}{R_0} = \alpha\ \Delta T$$

or

$$R(T)\ =\ R_0[1+\alpha\ (T-T_0)]$$

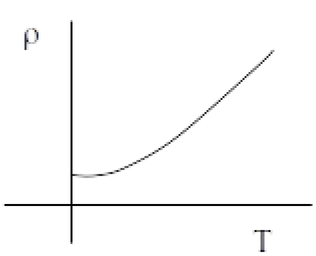
Since R and ρ are proportional, then we also have

$$\rho(T)\ =\ \rho_0[1+\alpha\ (T-T_0)]$$

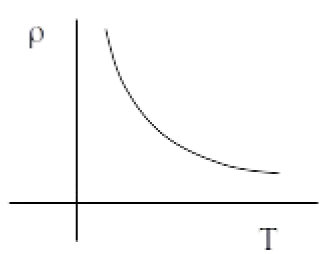
where α is the *temperature coefficient of resistivity* (not to be confused with the temperature coefficient of linear expansion) and T_0 = 20 degree Celsius.

For metals α is positive and is nearly constant over a large temperature range except for low temperatures. For semiconductors α is negative and varies strongly with temperature.

typical metal



semiconductor



In metals, the increase in ρ with increasing temperature is due to the scattering by atoms whose vibrational amplitude *increases* with temperature. That, is the time between collisions τ decreases with increasing temperature. In semiconductors, the decrease in ρ with increasing temperature is primarily due to the increasing number of conduction electrons (and holes).

In superconductors the resistivity drops abruptly to zero below a critical temperature. Lead is a superconductor with a critical temperature of 7.2 K. In 1987 a class of oxide superconductors was discovered with much higher transition temperatures. Superconducting electromagnets have wires made of superconducting alloys to enable the generation of large magnetic fields without excessive heating of the wires. These magnets require the use of liquid helium (boiling point = 4.2 K) to cool the current-carrying wires below the critical temperature.

Electrical Energy and Power Dissipation

In a circuit, a battery or power supply provides the potential difference to drive a current through the circuit. The infinitesimal work done by the battery to move an infinitesimal charge *dq* from the positive to the negative terminal is

$$dW = \Delta V\, dq$$

where ΔV is the constant potential difference provided by the battery. The total work is

$$W = \int \Delta V\, dq = \Delta V\, \Delta Q \quad \text{where} \quad \Delta Q = \int dq \quad \text{is the total charge}$$

The power supplied by the battery or power supply is the rate at which it does work during an infinitesimal time interval *dt*

$$P = \frac{dW}{dt} = \Delta V \frac{dq}{dt} = \Delta V\, I$$

Using $\Delta V = IR$, we can also write the power as

$$P = I^2 R = \frac{\Delta V^2}{R} \ .$$

Example

A 100-W light bulb and a 60-W bulb operate at a voltage of 120 V. What is the resistance of the bulbs?

For the 100-W bulb,

$$R = \frac{\Delta V^2}{P} = \frac{(120\,V)^2}{100\ W} = 144\ \Omega\ .$$

For the 60-W bulb,

$$R = \frac{(120\,V)^2}{60\ W} = 240\ \Omega$$

So the larger wattage bulb has the smaller resistance.

Example

What is the cost of keeping a 100-W bulb on for 1 month?

Alabama Power charges residents $0.105 per kwh of electrical energy usage.

1 kwh = 1 kilowatt-hour = (1000 J/s)(3600 s) = 3.6×10^6 J

The energy used in 30 days is

W = P · t = (0.1 kW)(30x24 hr) = 72 kwh

Cost = Rate x W = ($0.15/kwh)(72 kwh) = $7.56

Now let's see …..

If you leave a bunch of lights on all the time, say ten 60-W bulbs, and a PC (150 W), a and a TV (250 W), then you are using 1 kW, which would cost you $75.60 per month. And this doesn't include the big users such as your refrigerator, washer, dryer, electric water heater, air conditioner …