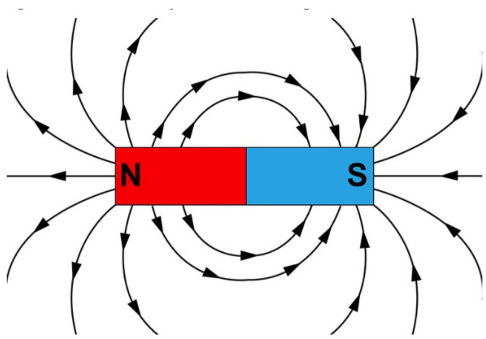
Magnetism

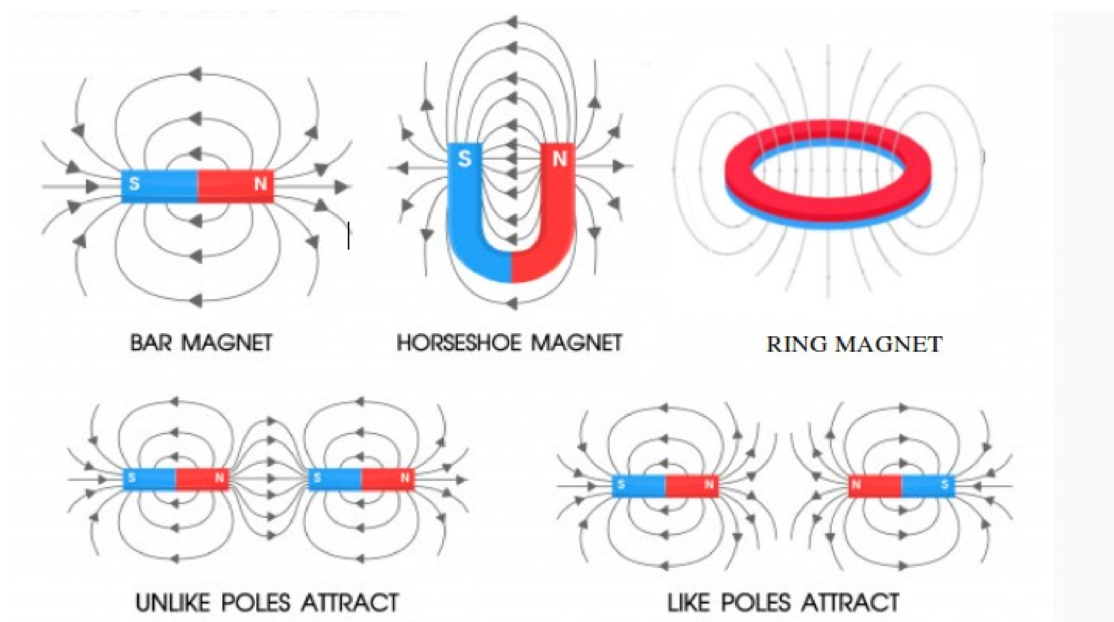
The Magnetic Field *B*

We most often associate magnetism with permanent magnets (later we will learn about electromagnets). The space surrounding these magnets can be described in terms of the magnetic field **B**, somewhat like the electric field in the space surround electric charges. All magnets have north and south poles. For an isolated magnet, such as the bar magnet shown below, the field lines emerge from the north pole and return to the south pole. All poles come in pairs. If you were to break the bar magnet into two pieces, you would create new north and south poles.



If you have two different magnets, their opposite poles attract and their like poles repel, somewhat like charges.

Examples

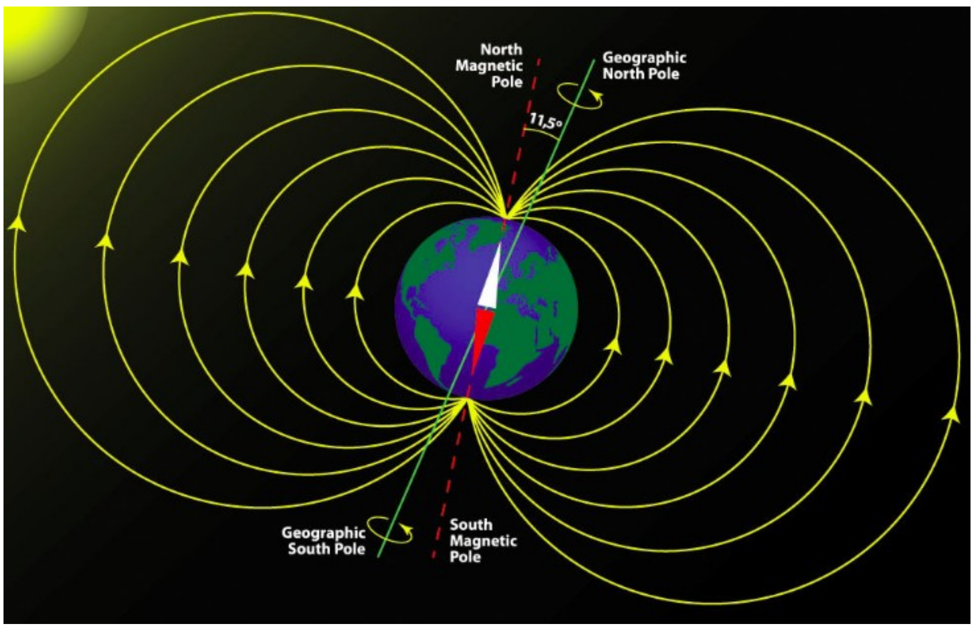


Units of B

The units of the magnetic field **B** is the Tesla, symbol *T*, where $1T = \frac{1N}{1A\cdot m}$. See the following paragraph on Lorentz force for this definition.
The unit of Tesla is usually too large for ordinary magnitudes of **B,** therefore it is often preferred the unit Gauss, symbol *G*, where $1G = 10^4 T$.

Earth's Magnetic Field

There is a magnetic field associated with the Earth, as the entire planet were a huge magnet.



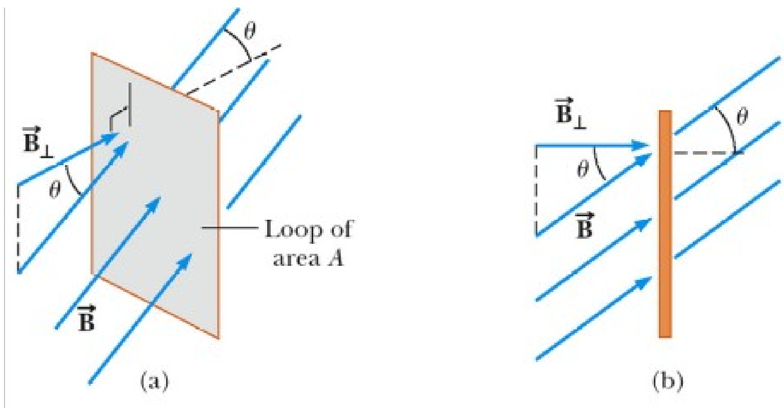
The magnetic poles do not exactly correspond to the geographic poles (rotational axis) but they are about 11.5 degree off. The north pole of a compass needle, which is a bar magnet, points north since it is attracted a 'south polarity'. This means that the magnetic pole located in the norther hemisphere is actually a south magnetic pole. It's refereed as 'north magnetic pole' because of its geographical location, not its polarity.

The magnitude of the Earth magnetic field depends on the location and it varies in the range from 25 to 65 micro Tesla (or .25 to .65 Gauss).

Magnetic Flux

The definition of magnetic flux is similar to the definition of electric flux. The flux through a surface depends on the number of field lines that go through the surface.

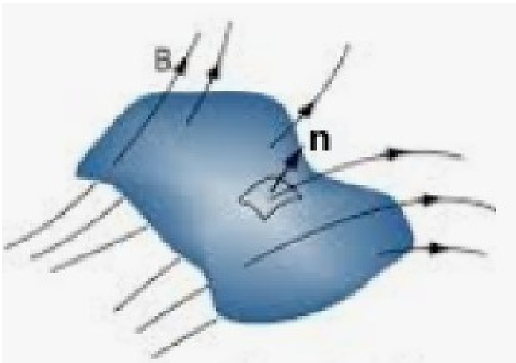
$\Phi(B) = BAcos\theta$ unit = T m^2 = weber (Wb)



$B_{\perp}$= B cosθ is the component of B that is perpendicular to the area A.

In the more general case of **B** or θ varying over the area, the flux is

$$\Phi(\vec{B}) = \int \vec{B} \cdot d\vec{A}$$



with

$$d\vec{A} = \hat{n} \cdot dA$$

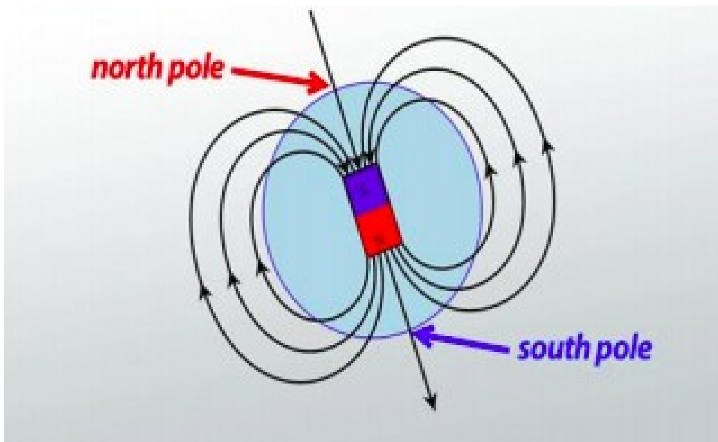
and **n** is the unit normal to *dA.*

Gauss Law for Magnetism

If the magnetic flux is calculated over a closed surface then we have the Gauss Law for magnetism

$$\Phi\left(\vec{B}\right) = 0$$

The magnetic flux through any closed surface is always zero. The reason is because magnetic poles always exist in the north-south pair, i.e. no monopoles exist in nature.

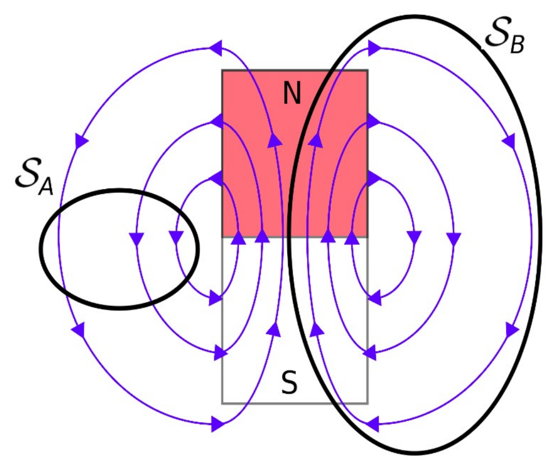


The magnetic flux through surface S_A is zero since all the flux lines which enter, also leave.

The magnetic flux through surface S_B is zero since none of the flux lines enter or leave.

Example

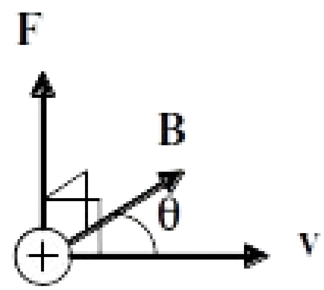
Try to draw on the magnet a Gaussian *Sc* such that the number of incoming field lines is different from the outgoing field lines: it is not possible.



The fundamental importance of the Gauss law for magnetism is that <u>it is the second</u> of the Maxwell Equations.

The Lorentz Force

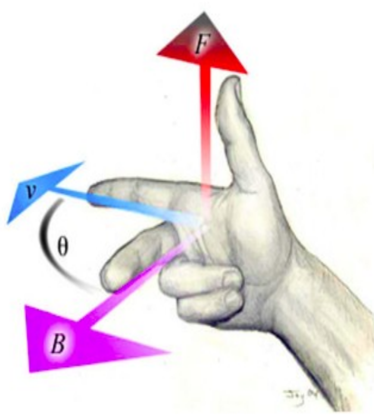
We can define the magnetic field by the force that it exerts on a charge. Unlike the electric field **E**, a magnetic field will not exert a force on an electric charge unless it is moving. The magnitude of the force is

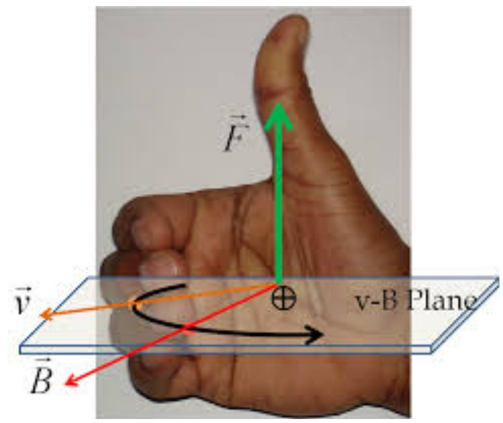


$$F = qvB\sin\theta$$

where θ is the angle between the direction of the field and the particle's velocity, i.e. it is only the component of field that is perpendicular to the velocity that matters.

The direction of the force is such that it perpendicular to both **B** and **v**, as shown. The direction of **F** can be determined by the 'right hand rule'. You point your fingers in the direction of **v**, then close you hand so that your fingers turn towards **B**, and your thumb is then in the direction of **F**.





For negative charges the force is in the opposite direction. Use the right hand rule and then reverse directions (or use a 'left hand rule'). Using vector notation, the Lorentz force is given by the cross product

$$\vec{F} = q(\vec{v}\times\vec{B})$$

This force is also referred as the magnetic force.

The Lorentz force equation is used to define the magnetic field operationally (by taking experimental measurements). So we could write

$$B = \frac{F}{qv\sin\theta}$$

The SI unit for B is Tesla (T). Another unit that is used is Gauss (G). 1 G = 10^{-4} T.

Example

A proton travels at a speed of 5 x 10^5 m/s in a direction 30° east of north in a region where the earth's magnetic field is 3 x 10^{-5} T and points north. What is the magnitude and direction of the force on the electron?

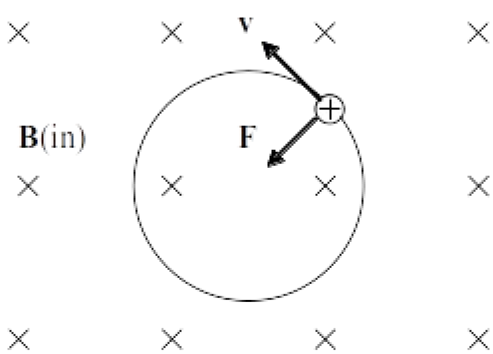
From the right hand rule, the direction of **F** is up. The magnitude is

$$F = (1.6 x 10^{-19} C)(5 x 10^5 m/s)(3 x 10^{-5} T)\sin(30^o) = 1.2 x 10^{-18} N$$

Although this may seem small, the resulting acceleration of the proton is huge. (Do the calculation.)

<u>Motion of a charged particle in a magnetic field</u>

A charged particle moving perpendicular to a magnetic field will experience a force given by $F = qvB$. Since this force is always perpendicular to **v**, then it will cause the particle to move in a circle. Also, since the force is perpendicular to **v** it does no work on the charge, and the kinetic energy and the speed of the particle don't change. Thus, we have uniform circular motion. So,



$$F = qvB = m\frac{v^2}{r},$$

$$r = \frac{mv}{qB}$$

Example

What is the radius of the circular motion of an electron moving at 10^6 m/s in a 0.05 T magnetic field?

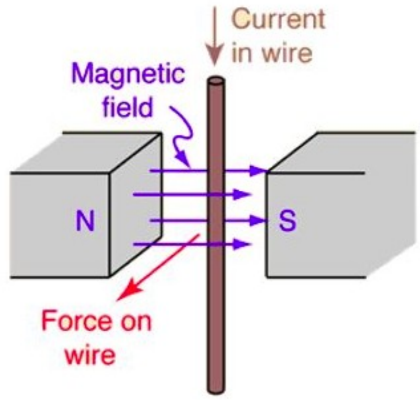
$$r = \frac{mv}{qB} = \frac{(9.1 x 10^{-31} kg)(10^6 m/s)}{(1.6 x 10^{-19} C)(0.05 T)} = 1.1 x 10^{-4} m = 0.11\ mm$$

A proton with the same speed in the same field would move in a circle with radius 21 cm.

The proton and electron would circulate in opposite directions in the field.

Magnetic force on a current carrying wire

A wire carrying an electric current consists of moving charges. Therefore, a magnetic field will exert a force on a current carrying wire. The amount of moving charge in a section of a wire of length dx is $dQ = Idt$ with drift velocity is $v_d = dx/dt$. Thus, the infinitesimal force acting on dQ is



$$dF = dQ\, v_d\, B\sin\alpha = Idt\frac{dx}{dt}B\sin\alpha = I\,dx\,B\sin\alpha$$

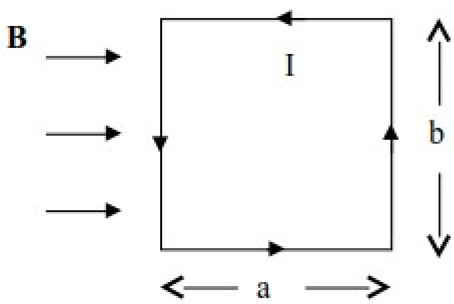
Adding up all the infinitesimal forces acting on the entire length $L = \Delta x$ of a straight wire, we obtain the force acting on the wire as $F=\int dF=\int IB\sin\alpha\ \ dx\ = \ IB\sin\alpha L$, or

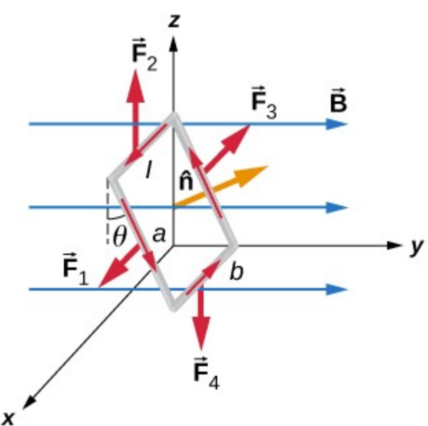
$$\vec{F} = L(\vec{I}\times\vec{B})$$

where α is the angle between the current $\boldsymbol{I}$ and **B**.
Again, we use the right hand rule to find the direction of **F**. It is perpendicular to both the wire and the field.

Torque on a current loop

A current loop in a magnetic field can experience forces on its sides which can generate a torque. Consider a rectangular loop with the field in the plane of the loop.



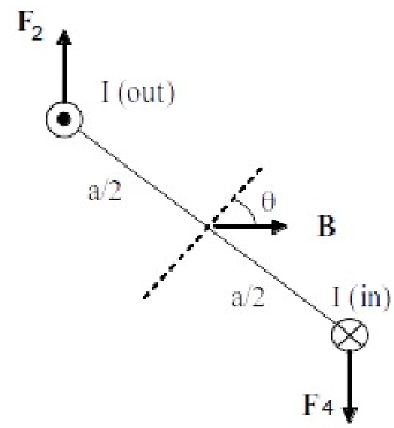


In the figure above on the left, the force F_2 is out of the page and F_4 *is* into the page. Since the current is parallel or antiparallel to the **B** field the forces F_1 and F_3 are zero. The two forces F_2 and F_4 create a torque which rotates the loop about a vertical axis through the center of the loop
In the figure above on the right, the force on the left side F_2 is up and the force on the right side F_4 is down, while F_1 and F_3 are non zero but being antiparallel don't contribute to the total torque. The max torque (θ = 90°) about the axis is

$$\tau = \tau(left) + \tau(right) = F\frac{a}{2} + F\frac{a}{2} = Fa$$

$$F = ILB = IbB, \; so$$

$$\tau = IabB = IAB$$



where $A = ab$ is the area of the loop.

If the field is not in the plane of the loop, then the torque is reduced by a factor sin θ. In particular, if the field is perpendicular to the plane then the torque is zero. To the right is a side view of the loop when the loop normal is at angle θ with respect to the field. The torque is

$$\tau = IAB\sin\theta$$

If the loop has N turns of wire, then the torque is30^{o}.

$$\tau = NIAB\sin\theta \qquad \text{(unit = Nm)}$$

This expression for the torque, although derived for a rectangular loop, applies to any loop shape.

The term *NIA* is called the magnetic dipole moment (or magnetic moment) of the loop

$$\mu = NIA \qquad \text{(unit = Am}^2\text{)}$$

So, we can write $\tau = \mu\, B\sin\theta$ or

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

where the direction of μ is given by the normal to the surface. The magnetic moment is useful to explain the microscopic magnetic proprieties (see next chapter).

Example

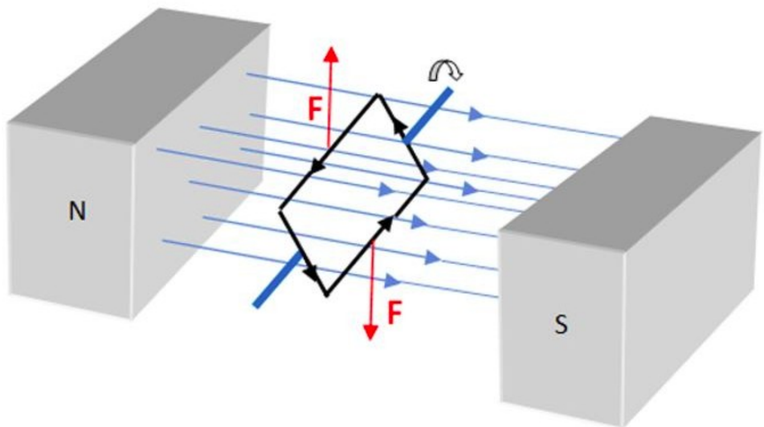
A 20-turn circular loop of wire of radius 4 cm carrying a current of 5 A is placed in a magnetic field of 0.1 T. The angle between the field direction and the plane of the loop is 30^{o}. Find the torque on the loop.

The angle between the *normal* to the plane and the field direction is 90^{o} – 30^{o} = 60^{o}. So,

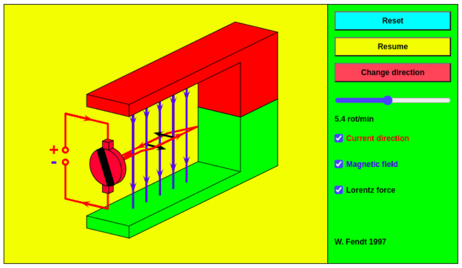
$$\tau = NIAB\sin\theta = (20)(5\,A)\,\pi\,(0.02m)^2(0.1\,T)\sin 60^o = 0.011\ N\cdot m$$

Electric motor

The electric motor is a device which makes use of the torque exerted on a current in a magnetic field to convert electrical energy into mechanical energy.
A commutator reverses the current direction each half rotation to keep the motor continuously rotating in the same direction. Otherwise, it would just oscillate back and forth.

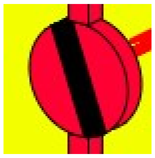


An animation can be found here:
https://www.walter-fendt.de/html5/phen/electricmotor_en.htm



Notice the directions of the *I*,*B* and the magnetic force *F*. T

The direction of the current in the wire needs to be inverted each 180 degree order of maintaining the rotation. The *commutator* serves this purpose, without it the coil would oscillate back and forth



the commutator