Chapter 14 - Fluids

This chapter deals with fluids, which means a liquid or a gas. It covers both fluid statics (fluids at rest) and fluid dynamics (fluids in motion).

Pressure in a fluid

Pressure is defined as the perpendicular force on a surface per unit surface area.

\[ P = \frac{F}{A} \quad [N/m^2 = \text{pascal} = \text{Pa}] \]

Mass density is mass per unit volume.

\[ \rho = \frac{M}{V} \quad [\text{kg/m}^3, \text{g/cm}^3, \ldots] \]

Water has a density of about 1 g/cm³ (= 10³ kg/m³).

Due to gravity, pressure increases with depth in a fluid as

\[ P = P_0 + \rho gh \]

where \( P_0 \) is the pressure at the top of the fluid and \( P \) is the pressure at a depth \( h \). As seen in the diagram to the right, additional pressure below a section of a fluid is required to keep this section from sinking. Since this section of fluid is in equilibrium,

\[ PA - P_0 A - Mg = 0 \]

But \( M = \rho V = \rho Ah \), so

\[ PA - P_0 A - \rho Ahg = 0 \]
\[ P = P_0 + \rho gh \]

If the fluid container is open to the atmosphere, then \( P_0 \) is atmospheric pressure. At sea level

\[ P_0 = 1.013 \times 10^5 \text{ Pa} \quad (= 14.7 \text{ lb/in}^2 = 1 \text{ atm}) \]
Example:

What is the downward force exerted by the atmosphere on top of a 2 m x 1 m desk top.

\[ F = P_0A = (1.013 \times 10^5 \text{ Pa})(2 \text{ m}^2) = 2.026 \times 10^5 \text{ N} \]

Or,

\[ F = (2.026 \times 10^5 \text{ N})(0.225 \text{ lb/N}) = 45,585 \text{ lb} \]
\[ = (45,585 \text{ lb})(1 \text{ ton/2000 lb}) = 22.8 \text{ tons!!} \]

The reason why this enormous force doesn’t crush the desk is because of a nearly equal upward force on the bottom of the table.

Example:

At what depth in water is the pressure 2 atm?

\[ P = 2P_0 = P_0 + \rho gh \]
\[ \rho gh = P_0 \]
\[ h = \frac{P_0}{\rho g} = \frac{1.013 \times 10^5 \text{ Pa}}{(10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 10.3 \text{ m} \]

Example:

A mercury manometer consists of an inverted tube of mercury as shown to the right. The top end is closed and the void at the top is essentially a vacuum. The bottom end is open and is in an open container of mercury. What is the height of the column of mercury in the tube? The specific gravity of mercury is 13.6.

\[ P = P_0 + \rho gh \]
\[ P = P_{\text{atm}} \text{ (bottom)} \]
\[ P_0 = 0 \text{ (top)} \]
\[ h = \frac{P_{\text{atm}}}{\rho g} = \frac{1.013 \times 10^5 \text{ N/m}^2}{(13.6 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} \]
\[ h = 0.76 \text{ m} = 76 \text{ cm} \]
Hydraulic press

A hydraulic press uses a fluid to magnify an applied force. A force $F_1$ applied to the small piston of area $A_1$ increases the pressure in the fluid by $P_1 = F_1/A_1$. This pressure increase is transmitted uniformly throughout the fluid (Pascal’s principle). This additional pressure results in a lift on the large piston.

\[
P_2 = P_1 \\
\frac{F_2}{A_2} = \frac{F_1}{A_1} \\
F_2 = F_1 \frac{A_2}{A_1}
\]

Example:

In a hydraulic press, the diameter of the small piston is 2.5 cm and the diameter of the large piston is 10 cm. If the force applied to the small piston is 500 N, what is the force applied to the large piston?

\[
F_2 = F_1 \frac{A_2}{A_1} = (500 \text{ N}) \frac{\pi (0.05 \text{ m})^2}{\pi (0.0125 \text{ m})^2} = 8000 \text{ N}
\]

Is conservation of energy violated? Not really. The large piston only moves $1/16^{th}$ as far as the small piston, so the work done in pushing the two pistons is the same (in the absence of resistance).

Archimede’s Principle

*Archimede’s principle* states that an object submerged in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the object.

\[
B = W_f = m_f g = \rho_f V_{\text{obj}} g
\]

The buoyant force, $B$, is just a consequence of the fact that the pressure below the object is greater than above it. To understand Archimede’s principle, we envision replacing the object with fluid of the same size and shape. This fluid must be in equilibrium and have the same buoyant force as the object. Thus, its weight and the buoyant force must be the same.
Example:

A cubical block of aluminum 10 cm on edge is suspended in water by a cord. What is the tension in the cord? The density of Al is $2.7 \times 10^3$ kg/m$^3$.

Since the block is in equilibrium,

$$T + B - m_{Al} g = 0$$

$$T = m_{Al} g - B = m_{Al} g - m_{water} g$$

$$= \rho_{Al} V g - \rho_{water} V g = (\rho_{Al} - \rho_{water}) g V$$

$$= (2.7 \times 10^3 \text{ kg} / \text{m}^3 - 1 \times 10^3 \text{ kg} / \text{m}^3)(9.8 \text{ m} / \text{s}^2)(0.1 \text{ m})^3$$

$$= 16.7 N$$

Example:

An ice cube floats in a glass of water. What fraction of its volume is below water? The specific gravity of ice is 0.917.

$$W_{ice} = B$$

$$m_{ice} g = m_{water} g$$

$$\rho_{ice} V g = \rho_{water} V_{below} g$$

$$\frac{V_{below}}{V} = \frac{\rho_{ice}}{\rho_{water}} = 0.917$$

Thus, 91.7% of the volume of the ice is below the water.

**Fluid Dynamics**

**Equation of continuity**

The rate at which fluid mass flows through two different parts of the same pipe must be the same. Thus,

$$\Delta M_1 = \Delta M_2$$

$$\rho_1 \Delta V_1 = \rho_1 \Delta V_2$$

$$\rho_1 A_1 \Delta x_1 = \rho_1 A_2 \Delta x_2$$

$$\rho_1 A_1 v_1 \Delta t = \rho_1 A_2 v_2 \Delta t$$
If the fluid is *incompressible*, i.e., its density is nearly the same throughout the pipe, then we have

\[ A_1 v_1 = A_2 v_2 \]  
(Eq. of continuity)

**Example:**

A hose of diameter 3 cm has a nozzle of diameter 1 cm. If the water flows at 2 m/s in the hose, what is the water speed as it goes through the nozzle?

\[
v_2 = v_1 \frac{A_1}{A_2} = v_1 \frac{\pi (3)^2}{\pi (1)^2} = (2 \text{ m/s}) \frac{(3)^2}{(1)^2} = 18 \text{ m/s}
\]

Bernoulli’s equation:

Bernoulli’s equation gives a relationship in a flowing fluid between the fluid’s pressure, flow speed, and elevation. It is based on conservation of energy and holds for an ‘ideal’ fluid. The ideal fluid would be (1) non-viscous, (2) incompressible, (3) steady in its flow, and (4) non-turbulent.

\[
P + \frac{1}{2} \rho v^2 + \rho g y = \text{constant}
\]

Bernoulli’s equation

This means that if you pick any two points in a flowing fluid,

\[
P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2
\]

If the fluid is at rest \((v_1 = v_2 = 0)\), then Bernoulli’s equation is the same as the earlier equation giving \(P\) as a function of depth in a fluid –

\[
P_1 = P_2 + \rho g (y_2 - y_1)
\]

Qualitatively, Bernoulli’s equation says that the pressure is lower in a region of a fluid where its speed is greater.

**Example:**

An airplane wing has curvature and angle of attach such that the air speed above the wing is greater than below. If \(v(\text{below}) = 100 \text{ m/s}\) and \(v(\text{above}) = 105 \text{ m/s}\) and the area of the wing is 10 m², what is the lift on the wing? The density of air is about 1.3 kg/m³. The difference in elevation below and above the wing is nearly the same, so \(y_1 \sim y_2\) and
\[
\Delta P = P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) = \frac{1}{2} (1.3 \text{kg/m}^3)((105 \text{m/s})^2 - (100 \text{m/s})^2)
\]
\[= 666 \text{ N/m}^2
\]
\[Lift = F = \Delta PA = (666 \text{ N/m}^2)(10 \text{m}^2) = 6660 \text{ N}
\]

Example:

A liquid in a pressurized container has a hole in the side. What is the speed of the liquid coming out of the hole?

We apply Bernoulli’s equation to two points – the point where the fluid leaves the hole (point 1) and a point at the top of the fluid in the container (point 2).

\[P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2
\]

The pressure at the hole is atmospheric pressure and the pressure, \(P_1 = P_a\), and the pressure at the top of the fluid is \(P_2 = P + P_a\). Also, we assume that volume of the fluid is such that \(v_2 \approx 0\). Then,

\[P_a + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P + P_a + \rho g y_2
\]

Or, using \(h = y_2 - y_1\), we get

\[v_1 = \sqrt{\frac{2P}{\rho} + 2gh}
\]

\(P\) is the ‘gauge’ pressure at the top of the fluid, i.e., the pressure above atmospheric pressure.

If the container is open, then \(P = 0\) and

\[v_1 = \sqrt{2gh}
\]

This is just the speed that the fluid would obtain if it fell directly a height \(h\).

**Question:** Can you calculate how far the fluid leaving the hole would travel horizontally before hitting the ground?
Wind energy

How much power can be generated by a wind turbine?

The kinetic energy carried by the wind per unit volume is

$$\frac{KE}{volume} = \frac{1}{2} \rho v^2$$

If all this kinetic energy could be converted into power, then the power would be

$$Power = \frac{KE}{volume} \times \frac{volume}{time} = (\frac{1}{2} \rho v^2)(Av) = \frac{1}{2} \rho v^3 A$$

Or, the power per unit area is

$$\frac{Power}{A} = \frac{1}{2} \rho v^3$$

Thus, the power available varies with the cube of the windspeed. If the windspeed doubles, then the power increases by a factor of 8. If the windspeed increases by 25%, then the power increases by about a factor of 2. So, windy regions are especially important for wind power.

Not all of the power given by the above equation can be extracted from the wind. For one thing, this assumes that the wind would be brought completely to rest by the turbine. It also neglects the efficiency of conversion into electricity. The actual power that can be extracted from the wind no more than about 15% of that given by the equation.