

ERROR ANALYSIS EXPERIMENT

Introduction

In this experiment, we are interested in learning how to treat data. Any time a scientist measures a number; he/she must be able to tell his colleagues the accuracy of that number. Otherwise, there is no way to tell whether the number agrees with the predictions of a theory; there is also no way for another scientist to check the experiment.

For example, consider the ratio of the circumference C and the diameter D of a circle,

$$\frac{C}{D} = \frac{2\pi r}{2r} = \pi$$

Suppose I measure the circumference and the diameter of a circle and find their ratio to be 3.15, did I just prove that π is not equal to 3.1415..?

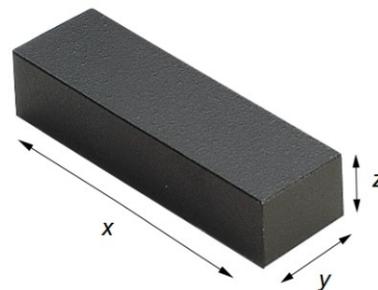
Equipment

Vertical rod, clamp, hook-rod, string (at least 1 m), mass hanger, mass (100 g), metal block.

PART 1 - Reading Scale

Use the ruler to measure the length of the three sides of the metal block on the table.

| X-side (mm) | Y-side (mm) | Z-side (mm) |
|-------------|-------------|-------------|
| | | |



The length obtained from a single measurement can be written as,

$$L = L_0 \pm \delta L$$

Where L_0 is the value measured for any of the sides (X , Y or Z) and δL is the error in each of these measurements. δL is defined by the accuracy of the instrument used.

1. What is δL of the ruler (the smallest marked interval)?

$$\delta L = \underline{\hspace{2cm}}$$

2. Choose one face of the metal block and calculate its perimeter, $P_0 = 2X+2Y$

$$P_0 = \underline{\hspace{2cm}}$$

Make a prediction, will the error be the same/greater/smaller than δL ?

The correct way to calculate the error of an arbitrary sum of two independent variables, $A = a X + b Y$ is,

$$\delta A = \sqrt{a^2 \cdot (\delta X)^2 + b^2 \cdot (\delta Y)^2}$$

In the case of the perimeter P the error in the measurement δP is given by

$$\delta P = \sqrt{4 \cdot (\delta X)^2 + 4 \cdot (\delta Y)^2}$$

Use the previous equation to calculate δP , where in our case $\delta X = \delta Y = \delta L$

$$\delta P = \underline{\hspace{2cm}}$$

Altogether, the final answer for our measurement is

$$P = P_0 \pm \delta P = \underline{\hspace{1cm}} \pm \underline{\hspace{1cm}}$$

3. Calculate the volume of the metal block $V_0 = X Y Z$

$$V_0 = \underline{\hspace{2cm}}$$

Make a prediction, would the error be the same/greater/smaller than three times δL ?

The correct way to calculate the error for the product of three independent variables is,

$$\delta V = V_0 \sqrt{\left(\frac{\delta X}{X}\right)^2 + \left(\frac{\delta Y}{Y}\right)^2 + \left(\frac{\delta Z}{Z}\right)^2}$$

Use the previous equation to calculate δV . In our case $\delta X = \delta Y = \delta Z = \delta L$

$$\delta V = \underline{\hspace{2cm}}$$

Altogether, the final answer for our measurement would be

$$V = V_0 \pm \delta V = \underline{\hspace{1cm}} \pm \underline{\hspace{1cm}}$$

PART 2 – Average and Standard deviation

In this part of the lab you will measure the period of oscillation, T , of a pendulum. Use the computer to find a webpage with a stopwatch, or use your phone. The stopwatch should have accuracy of at least ± 0.01 s. There is a difference with PART 1: now we have a more significant source of error.

4. What would it be?

Step 1. Displace the pendulum 15 degrees or less from its resting position.

Step 2. Using the stopwatch, measure T_i twenty times (with accuracy of ± 0.01 s, round it if needed. Note: the stopwatch on your phone might have an accuracy of ± 0.1 s, which is not precise enough)

Step 3. Use the next table to collect your data.

Analysis

You will learn later in the semester that T depends only on the length of the pendulum and not on the hanging mass or how far away from the equilibrium position you displace the pendulum.

The average (or mean) value of the time period is calculate from the equation

$$\bar{T} = \frac{1}{N} \cdot \sum_{i=1}^N T_i$$

where N is the number of the measurements taken.

5. What is the average period?

$$\bar{T} = \underline{\hspace{2cm}}$$

Next, what is the error of our measurement? First, let's introduce d , the deviation from the average value:

$$d_i = T_i - \bar{T}$$

6. Calculate d_i for each of your measurements.

| Measurement number, i | Measurement value, T_i (s) | Deviation, d_i (s) |
|-------------------------|------------------------------|----------------------|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |
| 11 | | |
| 12 | | |
| 13 | | |
| 14 | | |
| 15 | | |
| 16 | | |
| 17 | | |
| 18 | | |
| 19 | | |
| 20 | | |

We are interested in the average deviation from our best estimate. So should we just take the average of the d_i ?

7. Calculate the average of the deviation,

$$\bar{d} = \frac{1}{N} \cdot \sum_{i=1}^N d_i = \underline{\hspace{2cm}}$$

Clearly, the average of deviations cannot be used as the error estimate, since it gives zero (or very close to zero, depending on how you have rounded your digits). This is because the deviations with positive sign are canceled by the deviations with negative sign. In order to obtain an estimate of the error of the average σ_{average} we first need the standard deviation σ_T . It's a 'natural' measure of the statistical dispersion when the center of the data is measured about the average.

8. Calculate the standard deviation,

$$\sigma_T = \sqrt{\frac{1}{(N-1)} \cdot \sum_{i=1}^N d_i^2} = \underline{\hspace{2cm}}$$

The error for the average σ_{average} is related to the standard deviation σ_T by

$$\sigma_{\text{average}} = \frac{\sigma_T}{\sqrt{N}}$$

9. Altogether, the final answer for our measurement is

$$T = \bar{T} \pm \sigma_{\text{average}} = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}}$$

10. Looking at the definition of σ_{average} , what is one way we can make σ_{average} as small as possible?

PART 3 – Data Distribution

We now want to display the data in the form of a **histogram** to show the frequency distribution of your measured times. This will be somewhat like a histogram that shows the distribution of student grades on an exam where, for example, the histogram shows the number of grades in 10 point intervals, or bins, from 0 to 100.

We will use Excel to plot the histogram.

Step 1. Enter your 20 measurements of T_i in column A in Excel.

Step 2. In column B enter bin numbers.

These numbers will include the range of your measurements from low to high in increments of 0.05 s. For example, if your lowest T_i was 1.58 s and your highest T_i was 1.82 s, then your bin numbers would be 1.55 s, 1.60 s, ..., 1.80 s, 1.85 s. Excel will determine the number of values of T_i in each bin, where the bin number represents the largest value of T_i in each bin.

Step 3. Click the *Data* tab and then *Data Analysis*.

Step 4. Select *Histogram* from the menu.

Step 5. The *Input Range* is the range of cells representing the data (column A) and the *Bin Range* is column B.

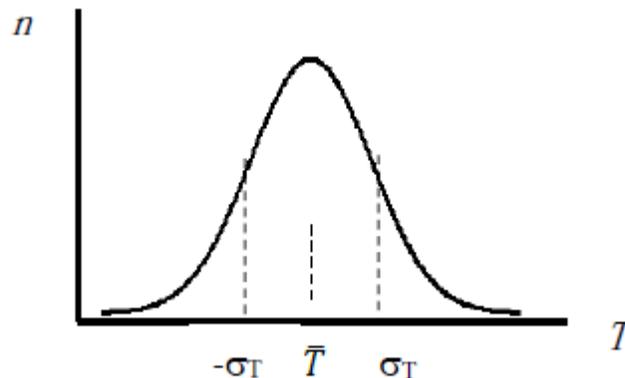
Step 6. Click *Chart Output* and then *OK* and Excel will plot the histogram of your data.

Step 7. Label your plots axes in in Excel.

Step 8. Select and print your histogram plot.

Analysis

Sketch a bell shaped curve on your graph that would correspond to a rough visual fit to the data. Ideally, if you took enough measurements this distribution would be given by a normal or Gaussian distribution function, like that shown below.



This curve helps to visualize the average value and the spread about the average.

11. How is the peak position of your curve related to the average value you calculated in question 5?

The Gaussian distribution also provides a graphical meaning for the standard deviation. In order to see this, print your graph (including plot labels on the axes).

12. Mark with a dot on the x -axis the average value of T . Mark also on the x -axis the two values of the standard deviation: $-\sigma_T$ on the left and $+\sigma_T$ on the right of the average of T . Draw two vertical lines (dotted lines) intersecting the x -axis at $\pm \sigma_T$ (σ_T : one standard deviation)

Does your graph look like the figure above?

13. How many of your data points lie between $\pm \sigma_T$? _____

14. Which percentage does it correspond to (the total number of data is $N = 20$) ? _____

Ideally, you should have obtained 68%. This 68% of values within $\pm \sigma_T$ is a mathematical property of the Gaussian distribution.

15. If you take the standard deviation as a measure of how well you can determine T_i in a single measurement, how does it agree with the initial guess of 0.01 s?