Special Relativity Formula

Kinematics

$$\Lambda^{\mu}{}_{\nu} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\gamma(v) = \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \frac{v}{c} \qquad v = \frac{v' + v_T}{1 + \frac{v' v_T}{c^2}}$$

$$\Delta L = \gamma^{-1} \Delta L_0 \qquad \Delta t = \gamma \Delta t_0$$

$$A'^{\mu} = \Lambda^{\mu}{}_{\nu} A^{\nu} \qquad A'_{\mu} = \Lambda_{\mu}{}^{\nu} A_{\nu}$$

$$A'^{\mu\nu} = \Lambda^{\mu}{}_{\alpha} \Lambda^{\nu}{}_{\beta} A^{\alpha\beta} \qquad A'_{\mu\nu} = \Lambda_{\mu}{}^{\alpha} \Lambda_{\nu}{}^{\beta} A_{\alpha\beta}$$

$$A_{\mu} = \eta_{\mu\nu} A^{\nu} \qquad A^{\mu} = \eta^{\mu\nu} A_{\nu}$$

$$x^{\mu} = (ct, x^i) \qquad ds^2 = \eta_{\mu\nu} dx^{\mu} dx^{\nu}$$

$$u^{\mu} = \frac{dx^{\mu}}{d\tau} = (\gamma c, \gamma v^i)$$

$A = A^{\mu}{}_{\mu} = \eta_{\mu\nu} A^{\mu\nu}$ (trace of $A^{\mu\nu}$)
$A^2 = A^{\mu} A_{\mu}$ (norm square of A^{μ})

Dynamics

$$p^{\mu} = m u^{\mu} = (E/c, p^i)$$

$$E = \gamma m c^2 \qquad p^i = \gamma m v^i \qquad KE = (\gamma - 1) m c^2 \qquad E_0 = mc^2$$

$$p^{\mu} p_{\mu} = m^2 c^2 \qquad p^{\mu}{}_{i} = p^{\mu}{}_{f}$$

Electromagnetism

$$\partial_{\mu} F^{\mu\nu} = \tfrac{4\pi}{c} J^{\nu} \qquad \partial_{[\mu} F^{\nu\rho]} = 0 \qquad F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$F^{\mu} = q F^{\mu}{}_{\nu} u^{\nu} \qquad J^{\mu} = \rho_0 u^{\mu} \qquad A_{\mu} = (c\varphi, A_i)$$