

SHOW all your works. Put the answers in a BOX NAME: _____

1 The nonzero components of the Ricci tensors of the flat ($\kappa = 0$) RW metric are

$$R_{00} = -3\frac{\ddot{a}}{a}, \quad R_{ij} = (a\ddot{a} + 2\dot{a}^2)\delta_{ij}$$

Calculate the Ricci scalar.

2 Using the Friedman equations and $\rho(t) = \rho_0(\frac{a_0}{a})^{3(1+\omega)}$ show that the age of the universe when written as function of ω and H_0 is $t_0 = \frac{2}{3(\omega+1)}H_0^{-1}$. Use $G = c = a_0 = 1$.

3 Suppose the scale factor is $a(t) = (t/t_0)^{1/3}$ where $t_0 = 13.6 \times 10^9$ years is the present age of the universe. What would it be the value of the Hubble constant today? (years⁻¹).

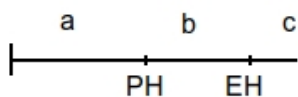
4 A galaxy observed today has cosmological redshift $z = 5.5$. In a universe with $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$ and $H_0 = 70$ km/(Mpc s), calculate:

4.1 the distance traveled by the light emitted from the galaxy toward us. (light years).

4.2 the recession velocity of the galaxy. (m/s).

5 The energy momentum tensor for a perfect fluid is $T_{\mu\nu} = (\rho_M - P/c^2)u_\mu u_\nu - P g_{\mu\nu}$ where ρ_M is the mass density and P the pressure. Show that in comoving coordinates it simplifies to $T_{00} = \rho, T_{ij} = P g_{ij}$ where ρ is the energy density.

6 In the past the event horizon of our universe was greater than the particles (or physical) horizon. You are the observer all the way to the left ($r = 0$). In which region a, b, c or *none* a galaxy was located in the past in order to satisfy the conditions described on the right of the figure?



6.1 We can't see the galaxy now and will not see it in the future.

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7 In a toy model of a Universe $a(t) = \alpha t^2$ where α is a constant. A galaxy is measured to have recession velocity = $0.3 c$ at $t = 5$ years. What is the recession velocity of that galaxy at $t = 20$ years? (unit of c).