PH482 - HW2 Lorentz Tensors

SHOW all your works. Put the answers in a BOX NAME:_____

1 Given the following components of the four-vector A :

$$A^\mu = (-2, 3, 1, -1)$$

Compute its components A'^μ after the Lorentz boost $v_x = 0.91c$

2 Write how the following tensorial quantities transform after a Lorentz transformation. Use the appropriate Lorentz Λ matrix for each index.

$$A_\mu;\ A^\mu$$
$$B^{\mu\nu};\ B_{\mu\nu}$$
$$C^\mu{}_\nu$$
$$D^{\mu\nu}{}_\rho$$
$$E^\mu D_\mu$$
$$E^\mu D^\mu$$
$$F^\mu G_\mu H^\rho$$

3 Given the tensors $U^\mu = (\gamma c, \gamma v^i)$, $U_\mu = (\gamma c, -\gamma v^i)$. Calculate:

$$U^\mu U_\mu$$
$$U'^\mu U'_\mu$$
$$U^\mu U^\mu$$

4 Use the Lorentz transformations to derive the expressions of length contraction and time dilation.

5 Perform the explicit matrix multiplications of the component of the Lorentz matrix with its inverse to prove that $\Lambda^{-1}\Lambda = \mathbb{1}_{4\times4}$

6 Write down explicitly all the terms in the sums of the 2D expression $A^2{}_1 = B^2{}_\alpha C_1{}^\beta D^\alpha{}_\beta$