PH482 - HW3 The metric

SHOW all your works. Put the answers in a BOX NAME:____________________

1 Given the following components of the two Lorentz vectors A and B:

$$A^\mu = (-2, 0, 0, 1) \qquad B^\mu = (5, 0, 3, 4)$$

1.1 Compute $A - 5B$
1.2 Compute AB
1.3 Calculate the norm squared of A and of B and specify if it is timelike, lightlike or spacelike.

2 Prove that $(\Lambda^\mu{}_\nu)^{-1} = \Lambda_\nu{}^\mu$ by performing the explicit calculations of the components of the Lorentz matrix.

3 Find the length of the curve

$$x(\kappa) = 2\kappa \quad y(\kappa) = -\kappa^3 \qquad 0 < \kappa < 1/2$$

on a two dimensional space with metric

$$g_{ij} = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$$

You can use software to evaluate the integration.

4 Given the tensor A with components

$$A^i{}_j = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$

Find the components of $A'^i{}_j$ resulting of a rotation of 50 degrees of the Euclidean x, y axis.

5 Given the two-dimensional Minkowski metric

$$\eta_{ij} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Find four two-tensors $A^\mu, B^\mu, C^\mu, D^\mu$ such that each one is lightlike, has all components non-zero and points in a unique direction. Draw the tensors on a spacetime diagram.

6 Given the tensor A^i and the metric g_{ij} below, find A_i.

$$A^i = (-2, 3) \qquad g_{ij} = \begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix}$$

7 Show that the spacial component η_{11} of the metric tensor is invariant under a Lorentz transformation.

8 Calculate the trace of $A_{ij}, B^{ij}, C^i{}_j$ with respect to the metric η_{ij}.

$$A_{ij} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} \qquad B^{ij} = \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix} \qquad C^i{}_j = \begin{pmatrix} -1 & 0 \\ -1 & 3 \end{pmatrix} \qquad \eta_{ij} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$