PH482 - HW4 - Special Relativity: Dynamics

SHOW all your works. Put the answers in a BOX NAME:_____

1 The equations of motion of a point particle is:

$$\begin{aligned} x^0(\tau) &= \alpha(\tau - 1) \\ x^1(\tau) &= \beta\tau^2 \end{aligned}$$

find the value of τ, in terms of α and β, which corresponds to $v = c$.

2 Show that $\gamma m v^2 + \gamma^{-1} m c^2 = \gamma m c^2$.

3 A particle moves along the x-axis with 3-velocity (in natural units $c = 1$):

$$\frac{dx}{dt} = \frac{kt}{\sqrt{1 + k^2 t^2}} \qquad k = constant$$

3.1 Calculate the components of the four velocity.
3.2 Give the expression $\tau(t)$ of the proper time elapsed from $t_i = 0$ to $t_f = t$:
3.3 Does the particle 3-speed ever exceed the speed of light?

4 An electron is moving with kinetic energy of 1.264 MeV. What is its speed? (unit of c)

5 A particle has relativistic momentum 817 MeV/c and energy of 1125 MeV for observer O.
5.1 What is its rest energy for O? (MeV)
5.2 Observer O' in a different frame measures the momentum of the same particle to be 953 Mev/c. What is the corresponding energy of the particle for O'? (MeV)

6 Electrons are accelerated to high speed by a two stages machine. The first stage accelerates the electron from rest to 0.99 c. The second from 0.99 c to 0.999 c.
6.1 How much energy is needed to accelerate the electron in the first stage? (MeV)
6.2 for the second? (MeV)
6.3 for the electron to reach the speed c?

7 A meson (rest mass 135 MeV) moves at speed $v = \frac{c}{\sqrt{2}}$ in a direction at 45° to the x-axis. Work in natural units
7.1 Find the components of the four velocity.
7.2 Find the components of the four momentum.

8 A particle of mass M decays at rest into two particles of same mass m. Find the speed of each particle (as a function of c, M, m).

9 Observer O measures a particle of mass m moving in the x direction to have speed $v = 0.67\ c$, energy $E = 1418$ MeV and momentum $p = 950$ MeV/c. For an observer O', moving at speed $v_T = .15\ c$ in the x direction, find:
9.1 E' the energy of the particle as measured by O'.
9.2 p' the momentum of the particle as measured by O'.