PH482 - HW5 - Electromagnetism

SHOW all your works. Put the answers in a BOX NAME:_____

1 Give the electromagnetic field tensor $F^{\mu\nu}$ and four current J^{μ} show explicitly that $\partial_{\mu}F^{\mu\nu} = \frac{4\pi}{c}J^{\nu}$ corresponds to the Gauss Law $\nabla \cdot E = \frac{\rho}{\varepsilon_0}$ and the Ampere Law $\nabla \times B = \mu_0 J + \mu\varepsilon_0 \frac{\partial E}{\partial t}$.

2 Give the electromagnetic field tensor $F^{\mu\nu}$ show explicitly that $\partial_{[\mu}F^{\nu\rho]} = 0$ corresponds to the Gauss Law $\nabla \cdot B = 0$ and the Faraday Law $\nabla \times E = -\frac{\partial B}{\partial t}$.

3 Given $F^{\mu\nu}$, find its component F'^{10} after a Lorentz boost $v_x = .85c$

4 Prove that $\partial_{[\mu}F_{\nu\rho]} = 0$ is satisfied identically when using $F_{\mu\nu}$ expressed in terms of the four vector potential A^{μ}.

5 Given the electromagnetic field tensor $F^{\mu\nu}$ and the four velocity u^{μ}.
5.1 show explicitly that $F^{\mu} = qF^{\mu}{}_{\nu}u^{\nu}$ corresponds to the Lorentz force $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ for the case of small speed ($v << c$).
5.2 show what the time component $\mu = 0$ of $F^{\mu} = qF^{\mu}{}_{\nu}u^{\nu}$ corresponds to.