PH482 - HW6 - Tensors

SHOW all your works. Put the answers in a BOX NAME:________________________

1 Given the scalar field $\phi(x,y) = x^2 - 2y$, and the change of coordinates: $x' = \frac{1}{3}(x-1), y' = 2y - x$, find:
1.1 the expression of $\phi'(x', y')$.
1.2 show that $\phi(x,y) = \phi'(x', y')$ point-wise, for example at $(x,y) = (2,1)$.

2 Given the tensor

$$A^i = (-y, 2, 3x,)$$

and the change of coordinates: $x' = xz, y' = y^2 - 2x, z' = -x^2 y + z$, find A^i in the new coordinates system.

3 Given the tensor:

$$A_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & x^2 \end{pmatrix}$$

and the change of coordinates: $x' = \frac{2x}{y}, y' = \frac{y}{2}$, find A'_{ij} in the new coordinates system.

4 The components of the metric tensor are:

$$g_{ij} = \begin{pmatrix} y & 3x & 0 \\ 3x & z^2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

Find the norm squared of the vector field with components $A^i = (z^2, x, -1)$ at the point $P = (1, 0, -2)$.

5 Given the Euclidean metric δ_{ij} in Cartesian coordinates (x,y), find its expression in the polar coordinates (r, θ). Use the indices sum. Show your step by step calculations.

6 Given the tensor A^μ prove that its partial derivative $\partial_\mu A^\nu$ is not a tensor.

7 Calculate the four-divergence $\partial^\mu \partial_\mu \phi(x^\mu)$ of the scalar field $\phi(x^\mu) = 2tx^3 - yz^2 + 3txy^2$ $(c = 1)$. The metric is:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -4 \end{pmatrix}$$