$\boxed{\textbf{PH482 - HW7}}$ - Riemannian Geometry

SHOW all your works. Put the answers in a BOX   NAME:________________________

1 Given the curve $c(\lambda) = (2\lambda, 1, \lambda^2 - 3\lambda)$ on a 3D manifold
1.1 find the corresponding tangent vector A at point P $(\lambda = 0)$.
1.2 given the function $f = (x^2 + y^2 + z^2)$, evaluate $A(f)$ at $\lambda = 0.25$.

2 The metric on $\mathbb{R}^2$ expressed in polar coordinate is

$$ds^2 = dr^2 + r^2 d\phi^2$$

Derive the non zero components of the connection ${\Gamma^i}_{jk}$.

3 The metric on the unit two-sphere S^2 expressed in spherical coordinate is

$$ds^2 = d\theta^2 + \sin^2(\theta) d\phi^2$$

Derive the non zero components of the connection ${\Gamma^i}_{jk}$.

4 The tangent vector A on the unit sphere S^2 has components $A^\theta = \sin(\theta), A^\varphi = \sin^2(\theta)$. Calculate all four components of its covariant derivative.

5 Show that the equator $(\theta = \frac{\pi}{2})$ and that any meridian $(\varphi = \text{constant})$ are the geodesics of S^2.

6 For a 2D manifold with coordinates $x^i = (x, y)$, the nonzero coefficients of connection are given by ${\Gamma^1}_{12} = {\Gamma^1}_{21} = 2,\ {\Gamma^2}_{11} = 3(y-1),\ {\Gamma^2}_{22} = 3x$. A tangent vector A with components $A^i = (3, -2)$ at $\lambda = 0$ is parallel transported to $\lambda = 0.8$ along the curve $x(\lambda) = \lambda^2 - 1,\ y(\lambda) = \frac{1}{3}\lambda + 1$. Find the new components of A.

7 The metric on the two-sphere S^2 of radius R in spherical coordinate is

$$ds^2 = R^2 d\theta^2 + R^2 \sin^2\theta d\phi^2$$

Use the metric above to prove that the area of half surface of the sphere is $2\pi R^2$.