PH482 - HW8 - Curvature

SHOW all your works. Put the answers in a BOX NAME:____________________________

1 Which of the following 2D Manifolds has non-zero intrinsic curvature?
Torus, Sphere, Cylinder, Mobius strip, Klein bottle, Two-holed torus, Hyperbolic plane.

2 The only independent non-zero component of the Riemann tensor of S^2 is $R_{\theta\phi\theta\phi} = a^2 \sin^2\theta$ where a is the radius of S^2 which is fixed. Starting from the Riemann tensor derive $R = \frac{2}{a^2}$.

3 The metric oh the hyperbolic plane in polar coordinates is:

$$ds^2 = a^2 d\kappa^2 + a^2 \sinh^2 k \; d\phi^2$$

where κ is a parameter $-\infty < \kappa < \infty$ and a is fixed. The non-zero elements of the connections are:

$$\Gamma^{\kappa}{}_{\phi\phi} = -\cosh\kappa \sinh\kappa \qquad \Gamma^{\phi}{}_{\kappa\phi} = \coth\kappa$$

3.1 Calculate $R_{\phi\kappa\phi}{}^{\kappa}$. All others components of the Riemann tensor are zero.
3.2 Show that the Ricci scalar is $R = -\frac{2}{a^2}$.

4 Use the symmetry of the Riemann tensor $R_{\mu\nu\rho\sigma} = R_{\rho\sigma\mu\nu}$ to prove that the Ricci tensor is symmetric.

5 A cylinder of radius a is embedded in $\mathbb{R}^3$.
5.1 Derive the metric of the cylinder expressed in cylindrical coordinate starting from the Euclidean metric in $\mathbb{R}^3$.
5.2 Show the curvature of the cylinder is zero.

6 The following tensorial equations holds in special relativity. Write the corresponding equations in the presence of gravity . The quantity α is constant, $\phi(x^\mu)$ and $\psi(x^\mu)$ are scalar functions.

$$\begin{aligned} \alpha\partial_\mu B^{\mu\nu} &= \eta_{\rho\sigma} C^{\rho\sigma\nu} \\ \phi(x) C_{\mu\nu} B^{\nu} &= \alpha\partial_\mu \psi(x) \\ \phi(x)\partial^\mu\partial_\mu A^\nu &= \psi(x)\eta^{\nu\rho} K_\rho \end{aligned}$$