PH482 - HW9 - Black Hole

SHOW all your works. Put the answers in a BOX NAME:______________________

1 Show that if $T_{\mu\nu} = 0$ the Einstein equation reduces to $R_{\mu\nu} = 0$.

2 The mass density of Earth is $\rho_E = 5515$ kg/m^3.
2.1 Calculate the Schwarzschild radius for the Earth (m).
2.2 What minimal value should ρ_E be for the Earth to form a black hole? (kg/m^3)?

3 An observer at infinity sees a pulse of light moving radially with speed 0.8 c near a Schwarzschild black hole of mass $M = M_{SUN}$.
3.1 What is the value of the radial coordinate of pulse?
3.2 What is the physical distance between the pulse and the event horizon?

4 Alice is in free fall toward a black hole. What gravitational effects does she observe as she crosses the event horizon of the black hole? Explain your answer in a few words.
A - very high speed.
B - dilation of the time intervals measured by her clock.
C - contraction of the time intervals measured by her clock.
D - very strong gravitational field.
E - none.

5 If the Sun turns into a black hole (with same original mass of the Sun) which gravitational effect will cause to the Earth's orbit? Explain your answer in a few words.
A - the Earth would fly off the tangent.
B - the Earth would spiral inward toward the Sun.
C - the Earth would move radially toward the Sun.
D - the Earth would be teared apart by the strong gravitational field.
E - none, the Earth would keep its original orbit.

6 Consider the coordinate system (u, v, θ, ϕ) where $u = t - r_*$, $v = t + r_*$ and $r_* = r + r_{SW} \ln(r/r_{SW} - 1)$. Prove that in this coordinate system the Schwarzschild metric is

$$ds^2 = (1 - \frac{r_{SW}}{r})dudv - r^2 d\Omega^2$$

Hint: start by calculating the differential dr_*, du, dv.

7 Alice is at rest nearby a Schwarzschild black hole and sends a light pulse every 5 seconds radially to Bob who stays at infinity. Bob receives the pulses every 10 seconds. Where is Alice? (unit of r_{SW}).

8 Alice is at rest at $r_A = 2\, r_{SW}$ from a Schwarzschild black hole and sends a light signal to Bob every 9 seconds. Bob is also at rest, at r_B, and receives that signals every 11 seconds. What is r_B? (unit of r_{SW}).

9 From energy considerations it can be shown that for an object starting at rest which moves radially in free fall toward a Schwarzschild black hole

$$(1 - \frac{r_{SW}}{r})\frac{dt}{d\tau} = 1$$

If the object starts from the position $r = 4\, r_{SW}$ and the black hole has mass $M = 3M_{Sun}$, calculate how long it takes for the object to reach the singularity. Hint: express $d\tau$ as $f(r)dr$.